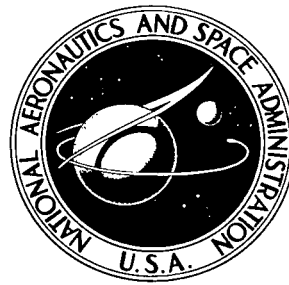


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ASTROMETRY AND ASTROPHYSICS

**No. 7 - Study of Errors and the Analysis
of Astrometrical Observational Results**

Edited by Ye. P. Fedorov

"Naukova Dumka" Press, Kiev, 1969

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1972



ASTROMETRY AND ASTROPHYSICS

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Edited by Ye. P. Fedorov

Translation of "Astrometriya i Astrofizika, 7,
Izucheniye oshibok i analiz rezul'tatov
astrometricheskikh nablyudeniye."
"Naukova Dumka" Press, Kiev, 1969

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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\$3.00

In this collection the problems of increasing the quality of astrometric observations and their processing are elucidated; instrumental errors are discussed, and design improvements to reduce them are discussed. A distinctive feature of the papers devoted to the analysis of the observations is the use of the methods of mathematical statistics to reveal hidden periodicities and a rigorous estimate of the reliability of the results. Also the problem of determining the zero points of the fundamental catalogues is discussed in detail.

This collection is intended for scientific workers, graduate students, and students engaged in the problems of fundamental and practical astrometry.

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ORIENTATION OF THE SYSTEMS OF FUNDAMENTAL CATALOGUES
FROM OBSERVATIONS OF MAJOR AND MINOR PLANETS

D. P. Duma¹

ABSTRACT: The author derives conditional equations for determining the fundamental catalogue zero-points from observation of major and minor planets. He discusses conditions under which corrections to equinox and equator could be separated from corrections to the elements of the planet's and earth's orbits. The conclusion is that determination of the corrections of the catalogue zero-points from observation of minor planets loses its advantage because of the complex correlations between the coefficients of the equations. Utilization of the stationary artificial earth's satellites is suggested to be considered as a possible method for the solution of the problem.

The main problem of fundamental astrometry is the construction of a system /3*
of coordinates which is fixed by a fundamental star catalogue of accurate positions with a high degree of accuracy of the known proper motions and precessional quantities of a definite number of stars uniformly distributed in the sky. All the quantities placed in the catalogue are referred to a single equinox, i.e., to the time of the start of some tropical year or other and to a single equator. The system of coordinates given by a fundamental catalogue should approximate as much as possible an ideal inertial system, i.e., not possessing motion except uniform and linear. The production of such a system of celestial coordinates is required for the solution of many problems in celestial mechanics and recently for the successful conquest of outer space.

One of the basic steps in the problem of determining the absolute right ascensions and declinations of stars is the determination of the position of the point of the vernal equinox and the fundamental equator for a given epoch.

The most widespread method of determining the corrections of a system of right ascensions and declinations of stars is Newcomb's method, which is applied

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* Numbers in the margin indicate pagination in the foreign text.

for observations of the sun and the major planets. Newcomb's equations are formulated for the sun and are very rigorous, if the theory of the solar motion is complete and the elements of the earth's orbit do not require corrections. We note that observations of the sun have already for a long time been processed by a more rigorous method which includes in the equation corrections for the elements of the earth's orbit [1-3, 30].

In connection with the fact that observations of the sun are carried out in the daytime and consequently contain systematic errors which are not inherent in nighttime observations of stars, the use of observations of the major planets was begun for the orientation of systems of coordinates of fundamental catalogues. The values of the inclinations of the orbits of the major planets are small; therefore it can be assumed that the planets move, as does the sun, in the plane of the ecliptic. If one considers the theory of motion of the planets as complete, then to improve the zero points of the catalogues successfully one can use old methods, for example, Newcomb's method, without any kind of changes in the fundamental equations. There still exists a series of other methods of determining ΔA and $\Delta \delta_0$. The main distinction of the latter from Newcomb's method is basically the inclusion in the equation as unknowns the flexure of the instrument, corrections to the refraction constant, and corrections to the elements of the orbits of the planets and others [1, 4, 5]. The use of these methods for the reduction of series of observations does not eliminate the discrepancy between individual values of the corrections of the equinox and equator. One of the main obstacles to obtaining values of the zero points which agree is assumed to be mainly observational errors, which it is not possible to exclude from the observed α and δ . They are particularly large for objects having disks.

At the start of the 20th century, observations of the moon, which because of its rapid motion among the stars has a number of advantages in comparison with the large planets, began to be used for improving the zero points of the fundamental catalogues. The first values of the corrections ΔA and $\Delta \delta_0$ were obtained by Newcomb's method [6]. It is impossible to consider their agreement very satisfactory, although it is better than for the planets. It is shown later that observations of the moon can be successfully used in determining the corrections of the equinox of the fundamental catalogue if during the reduction the asymmetry of the lunar disk is taken into account and more rigorous methods of determining ΔA and $\Delta \delta_0$ are used [7-12]. Experience shows that taking into

/4

account the libration effect in the radius and the corrections to the elements of the moon's orbit permits obtaining from short series (70-100 observations) agreement of the equinox corrections better than those obtained by other investigators who processed series ten times longer. However, at the present time it is not possible to recommend only the moon for orienting fundamental catalogues, since due to fundamental difficulties in determining the center of mass of the observations, the problem of determining the correction of the equator still remains unsolved. But due to the nearness of the moon to the earth, the large number of observations obtained earlier and being carried out at the present time, and also to the availability of data on the figure and relief, it is possible to solve this problem in a comparatively short time.

At the end of the 1930's it was proposed to use for orienting systems of fundamental catalogues observations of the minor planets. This idea was discussed in 1932 at the First Astrometric Conference at Pulkovo [13].

The question of the possibility and advisability of using the minor planets to determine the constants of catalogues is investigated in the papers [14-18]. In 1935 a plan of observation of the minor planets was proposed for the accomplishment of which work began in the Soviet Union on a large scale in 1955 in connection with the creation of the Catalogue of Faint Stars.^{*} Ten minor planets no fainter than $10^m.5$ at opposition were selected for the observations [24, 31].

A plan was worked out in 1935 for photographic observations of 14 minor planets in the USA [19]. Planets with various eccentricities and orbital inclinations were included in the list in order to investigate systematic errors of the star catalogues which vary not only with right ascension but also with declination in the zone $\pm 30^\circ$. It is suggested to introduce into the conditional equations 164 unknowns which characterize atmospheric dispersion, systematic errors of the star catalogue, and corrections to the elements of the orbits of the 14 planets. It is natural that such a large number of unknowns in the equations does not permit a reliable determination of the elements of orientation and the systematic errors of the catalogues. It seems to us that there is no necessity here to discuss in more detail the basic positions of these proposals and the methods of computing the zero points of the catalogues.

The first calculations of the corrections of the equinox and the equator from observations of Eros were carried out by Rabe in 1950 [20]. Following

^{*} Original title: Kataloga slabykh zvezd.

this, the results of processing observations of Nemausa [21] and Juno [22] were published, and recently - the results of a determination of the correction of the equinox and equator and also corrections to the elements of the orbits of the major planets Mars and Jupiter, and the minor planets Ceres, Pallas, Juno, and Vesta [23].

Rabe used in his derivation of the values of ΔA and $\Delta \delta_0$ more than 3000 observations of Eros over the period from 1926 to 1945, Naur used 750 observations of Nemausa from 1943 to 1954, and Orel'skaya used observations of Juno from 1934 to 1943 obtained at various observatories of the world both by visual and by photographic methods. See the paper [22] for a summary of the results of each author. The values of the elements of orientation obtained by Yasuda agree very poorly among themselves, and their errors of determination are very large, as in the papers [21, 22]. The total equinox correction of $+ 0^s.013$ agrees well /5 with the values of ΔA which we obtained from observations of the moon and which other authors obtained from observations of the major planets [25]. The poor agreement of the remaining values of the corrections to the zero points calculated from the observations of various objects give cause to return once again to theoretical discussions of the possibility of the inclusion of objects used up to now for the orientation of systems of fundamental catalogues. The goal of our subsequent investigations is to obtain rigorous conditional equations for the simultaneous determination of the errors of the catalogues and the corrections to the elements of the planet and the earth. We note that the equations obtained are valid for both the minor and the major planets because the former motions do not differ in nature from the latter.

The necessity of stating this problem is still dictated by the increase in accuracy of the observations. Consequently, more rigorous conditions are imposed for the accuracy of the determination of the zero points than 30-40 years ago. This situation requires a series of factors to be taken into account in processing the observations which were not considered earlier. The latter, in their turn, forced the creation of new methods of determining astronomical constants and the refinement of the old ones.

Derivation of Conditional Equations for the Determination of the Zero Points of Catalogues from Observations of the Major and Minor Planets

If differences are to be clarified between the calculated coordinates of a planet and those obtained from observations on the system of some catalogue or other, then they should be explained primarily by corrections to the elements of the orbits of the planet and the earth and also by the presence of systematic errors in the right ascensions and declinations of the catalogue used:

$$\begin{aligned}\Delta\alpha_0 &= d\alpha_0 + a + b \cos \alpha_0 + c \sin \alpha_0, \\ \Delta\delta_0 &= d\delta_0 + a' + b' \cos \alpha_0 + c' \sin \alpha_0.\end{aligned}\tag{1}$$

In addition to those indicated above, one can include in Eq. (1), as unknowns, the corrections to the masses of the major planets, coefficients of atmospheric dispersion, and others. In the present paper a limited problem is discussed - the possibility of determining 17 unknowns from observations of the major and minor planets from Eq. (1).

To obtain the differentials $d\alpha_0$ and $d\delta_0$ we used the well-known relations between the geocentric and heliocentric equatorial coordinates of a planet:

$$\begin{aligned}\rho \cos \delta_0 \cos \alpha_0 &= x - x', \\ \rho \cos \delta_0 \sin \alpha_0 &= y - y', \\ \rho \sin \delta_0 &= z - z',\end{aligned}\tag{2}$$

where

x, y, z and x', y', z' are the rectangular equatorial heliocentric coordinates of a planet and the earth;

ρ, α_0, δ_0 are the geocentric coordinates (spherical) of the planet.

Taking the differential of equations (2), we obtain the following relations:

$$\begin{aligned}\rho \cos \delta_0 d\alpha_0 &= (dy \cos \alpha_0 - dx \sin \alpha_0) - (dy' \cos \alpha_0 - dx' \sin \alpha_0), \\ \rho d\delta_0 &= dz \cos \delta_0 - \sin \delta_0 (dx \cos \alpha_0 + dy \sin \alpha_0) - dz' \cos \delta_0 + \\ &\quad + \sin \delta_0 (dx' \cos \alpha_0 + dy' \sin \alpha_0).\end{aligned}\tag{3}$$

Eq. (3) relates changes in the geocentric coordinates α_0 and δ_0 to changes in the heliocentric coordinates of the planet and the earth.

The rectangular heliocentric coordinates are presented in the following form:

$$\begin{aligned}
x &= r\alpha \sin(A+u), \\
y &= r\beta \sin(B+u), \\
z &= r\gamma \sin(C+u)
\end{aligned} \tag{4}$$

and

$$\begin{aligned}
x' &= r'\alpha' \sin(A'+u'), \\
y' &= r'\beta' \sin(B'+u'), \\
z' &= r'\gamma' \sin(C'+u'),
\end{aligned} \tag{5}$$

where

A, B, C, α , β , γ and A', B', C', α' , β' , γ' are the Gaussian constants for the planet and the earth, respectively.

After differentiation of the Eqs. (4) and (5) we have:

$$\begin{aligned}
dx &= (\alpha dr + r d\alpha) \sin(A+u) + d(A+u) r \alpha \cos(A+u), \\
dy &= (\beta dr + r d\beta) \sin(B+u) + d(B+u) r \beta \cos(B+u), \\
dz &= (\gamma dr + r d\gamma) \sin(C+u) + d(C+u) r \gamma \cos(C+u),
\end{aligned} \tag{6}$$

$$\begin{aligned}
dx' &= (\alpha' dr' + r' d\alpha') \sin(A'+u') + d(A'+u') r' \alpha' \cos(A'+u'), \\
dy' &= (\beta' dr' + r' d\beta') \sin(B'+u') + d(B'+u') r' \beta' \cos(B'+u'), \\
dz' &= (\gamma' dr' + r' d\gamma') \sin(C'+u') + d(C'+u') r' \gamma' \cos(C'+u').
\end{aligned} \tag{7}$$

Having substituted the values of dx, dy, dz, dx', dy', dz' into Eq. (3), we obtain the dependences of the differences of the right ascensions and declinations on the elements of the orbits of the planet and the earth:

$$\begin{aligned}
2\rho \cos \delta_0 d\alpha_0 &= [\beta \sin(B+u-\alpha_0) + \beta \sin(B+u+\alpha_0) - \\
&- \alpha \cos(A+u-\alpha_0) + \alpha \cos(A+u+\alpha_0)] dr + [\sin(B+u-\alpha_0) + \\
&+ \sin(B+u+\alpha_0)] r d\beta + [\cos(A+u+\alpha_0) - \cos(A+u-\alpha_0)] r d\alpha + \\
&+ [\cos(B+u-\alpha_0) + \cos(B+u+\alpha_0)] r \beta dB + [\sin(A+u-\alpha_0) - \\
&- \sin(A+u+\alpha_0)] r \alpha dA + [\beta \cos(B+u-\alpha_0) + \beta \cos(B+ \\
&+ u+\alpha_0) - \alpha \sin(A+u+\alpha_0) + \alpha \sin(A+u-\alpha_0)] r du - \\
&- [\beta' \sin(B'+u'-\alpha_0) + \beta' \sin(B'+u'+\alpha_0) - \alpha' \cos(A'+u'-\alpha_0) + \\
&+ \alpha' \cos(A'+u'+\alpha_0)] dr' - [\sin(B'+u'-\alpha_0) + \sin(B'+u'+ \\
&+ \alpha_0)] r' d\beta' - [\cos(A'+u'+\alpha_0) - \cos(A'+u'-\alpha_0)] r' d\alpha' - \\
&- [\cos(B'+u'-\alpha_0) + \cos(B'+u'+\alpha_0)] r' \beta' dB' - [\sin(A'+ \\
&+ u'-\alpha_0) - \sin(A'+u'+\alpha_0)] r' \alpha' dA' - [\beta' \cos(B'+u'-\alpha_0) + \\
&+ \beta' \cos(B'+u'+\alpha_0) - \alpha' \sin(A'+u'+\alpha_0) + \alpha' \sin(A' \\
&+ u'-\alpha_0)] r' du';
\end{aligned} \tag{8}$$

$$\begin{aligned}
2\rho d\delta_0 = & \sin \delta_0 [-\beta \cos (B+u-\alpha_0) + \beta \cos (B+u+\alpha_0) - \\
& -\alpha \sin (A+u-\alpha_0) - \alpha \sin (A+u+\alpha_0) + 2\gamma \sin (C+u) \operatorname{ctg} \delta_0] dr + \\
& + 2r \cos \delta_0 \sin (C+u) d\gamma + 2r\gamma \cos \delta_0 \cos (C+u) dC - \\
& - r \sin \delta_0 [\sin (A+u-\alpha_0) + \sin (A+u+\alpha_0)] d\alpha - \\
& - r\alpha \sin \delta_0 [\cos (A+u-\alpha_0) + \cos (A+u+\alpha_0)] dA - \\
& - r \sin \delta_0 [\cos (B+u-\alpha_0) - \cos (B+u+\alpha_0)] d\beta - r\beta \sin \delta_0 [\sin (B+ \\
& + u+\alpha_0) - \sin (B+u-\alpha_0)] dB + [2r\gamma \cos \delta_0 \cos (C+u) - \\
& - r\alpha \sin \delta_0 \cos (A+u-\alpha_0) - r\alpha \sin \delta_0 \cos (A+u+\alpha_0) - \\
& - r\beta \sin \delta_0 \sin (B+u+\alpha_0) + r\beta \sin \delta_0 \sin (B+u-\alpha_0)] du + \\
& + \sin \delta_0 [\alpha' \sin (A'+u'-\alpha_0) + \alpha' \sin (A'+u'+\alpha_0) + \\
& + \beta' \cos (B'+u'-\alpha_0) - \beta' \cos (B'+u'+\alpha_0) - 2\gamma' \sin (C'+ \\
& + u') \operatorname{ctg} \delta_0] dr' - 2r' \sin (C'+u') \cos \delta_0 d\gamma' - 2r'\gamma' \cos (C'+ \\
& + u') \cos \delta_0 dC' + r' \sin \delta_0 [\sin (A'+u'-\alpha_0) + \sin (A'+ \\
& + u'+\alpha_0)] d\alpha' + r'\alpha' \sin \delta_0 [\cos (A'+u'-\alpha_0) + \cos (A'+ \\
& + u'+\alpha_0)] dA' + r' \sin \delta_0 [\cos (B'+u'-\alpha_0) - \cos (B'+u'+ \\
& + \alpha_0)] d\beta' + r'\beta' \sin \delta_0 [\sin (B'+u'+\alpha_0) - \sin (B'+u'-\alpha_0)] dB' - \\
& - [2r'\gamma' \cos \delta_0 \cos (C'+u') - r'\alpha' \sin \delta_0 \cos (A'+u'-\alpha_0) - \\
& - r'\alpha' \sin \delta_0 \cos (A'+u'+\alpha_0) - r'\beta' \sin \delta_0 \sin (B'+u'+\alpha_0) + \\
& + r'\beta' \sin \delta_0 \sin (B'+u'-\alpha_0)] du'.
\end{aligned} \tag{9}$$

On the basis of Eqs. (2), (4), and (5) it is easy to write subsidiary relations with whose help in the coefficients of the Eqs. (8) and (9) one can separate the heliocentric and geocentric coordinates:

$$\begin{aligned}
\rho \cos \delta_0 \sin (A+u \pm \alpha_0) = & \frac{1}{2} [r\alpha - r\alpha \cos 2(A+u) - \\
& - r'\alpha' \cos (A'+A+u+u') \mp r\beta \sin (A-B) \pm r\beta \sin (A+B+ \\
& + 2u) \pm r'\beta' \sin (A-B'+u-u') \mp \\
& \mp r'\beta' \sin (A+B'+u+u') - r'\alpha' \cos (A-A'+u-u')]; \\
\rho \cos \delta_0 \cos (A+u \pm \alpha_0) = & \frac{1}{2} [r\alpha \sin 2(A+u) - \\
& - r'\alpha' \sin (A+A'+u+u') + r'\alpha' \sin (A-A'+u-u') \mp r\beta \cos (A- \\
& - B) \pm r\beta \cos (A+B+2u) \pm r'\beta' \cos (A-B'+u-u') \mp \\
& \mp r'\beta' \cos (A+B'+u+u')];
\end{aligned}$$

(10)

$$\begin{aligned}
\rho \cos \delta_0 \sin (B+u \pm \alpha_0) &= \frac{1}{2} [ra \cos (A-B) - ra \cos (A+B+ \\
&+ 2u) - r'\alpha' \cos (B-A'+u-u') + r'\alpha' \cos (A'+B+u+u') \pm \\
&\pm r\beta \sin 2(B+u) \pm r'\beta' \sin (B-B'+u-u') \mp r'\beta' \sin (B+B'+u+u')]; \\
\rho \cos \delta_0 \cos (B+u \pm \alpha_0) &= \frac{1}{2} [ra \sin (A-B) + ra \sin (A+B+ \\
&+ 2u) + r'\alpha' \sin (B-A'+u-u') - r'\alpha' \sin (A'+B+u+u') \mp \\
&\mp r\beta \pm r\beta \cos 2(B+u) \pm r'\beta' \cos (B-B'+u-u') \mp r'\beta' \cos (B+ \\
&+ B'+u+u')]; \\
\rho \cos \delta_0 \sin (A'+u' \pm \alpha_0) &= \frac{1}{2} [ra \cos (A-A'+u-u') - \\
&- ra \cos (A+A'+u+u') - r'\alpha' + r'\alpha' \cos 2(A'+u') \pm \\
&\pm r\beta \sin (B-A'+u-u') \pm r\beta \sin (B+A'+u+u') \mp \\
&\mp r'\beta' \sin (B'-A') \mp r'\beta' \sin (A'+B'+2u')]; \\
\rho \cos \delta_0 \cos (A'+u' \pm \alpha_0) &= \frac{1}{2} [ra \sin (A-A'+u-u') + \\
&+ ra \sin (A+A'+u+u') - r'\alpha' \sin 2(A'+u') \mp \\
&\mp r\beta \cos (B-A'+u-u') \pm r\beta \cos (B+A'+u+u') \pm \\
&\pm r'\beta' \cos (A'-B') \mp r'\beta' \cos (A'+B'+2u')]; \\
\rho \cos \delta_0 \sin (B'+u' \pm \alpha_0) &= \frac{1}{2} [ra \cos (A-B'+u-u') - \\
&- r'\alpha' \cos (A'-B') + r'\alpha' \cos (A'+B'+2u') \pm \\
&\pm r\beta \sin (B-B'+u-u') \pm r\beta \sin (B+B'+u+u') \mp \\
&\mp r'\beta' \sin 2(B'+u') - ra \cos (A+B'+u+u')]; \\
\rho \cos \delta_0 \cos (B'+u' \pm \alpha_0) &= \frac{1}{2} [ra \sin (A-B'+u-u') + \\
&+ ra \sin (A+B'+u+u') - r'\alpha' \sin (A'-B') - r'\alpha' \sin (A'+ \\
&+ B'+2u') \mp r\beta \cos (B-B'+u-u') \pm r\beta \cos (B+B'+u+u') \pm \\
&\pm r'\beta' \mp r'\beta' \cos 2(B'+u')].
\end{aligned}$$

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(11)

If we transform the Eqs. (8) and (9) with the help of the relations (10) and (11), then one can write the equations in such a form that the differentials $d\alpha_0$ and $d\delta_0$ are functions only of the heliocentric coordinates of the planet and the earth:

$$\begin{aligned}
2\rho^2 \cos^2 \delta_0 da_0 = & [\alpha\beta' \cos (A-B'+u'-u') - \alpha\beta' \cos (A+B'+u+u') - \\
& - \beta\alpha' \cos (B-A'+u-u') + \alpha'\beta' \cos (B+A'+u+u')] (r'dr - rdr') + \\
& + [r\alpha \cos (A-B) - r\alpha \cos (A+B+2u) - r'\alpha' \cos (B-A' + \\
& + u-u') + r'\alpha' \cos (A'+B+u+u')] r d\beta + [-r\beta \cos (A-B) + \\
& + r\beta \cos (A+B+2u) + r'\beta' \cos (A-B'+u-u') - r'\beta' \cos (A + \\
& + B'+u+u')] r d\alpha + [r\alpha \sin (A-B) + r\alpha \sin (A+B+2u) + \\
& + r'\alpha' \sin (B-A'+u-u') - r'\alpha' \sin (A'+B+u+u')] r\beta dB + \\
& + [r\beta \sin (A-B) - r\beta \sin (A+B+2u) - r'\beta' \sin (A-B' + \\
& + u-u') + r'\beta' \sin (A+B'+u+u')] r\alpha dA - [r\alpha \cos (A-B' + \\
& + u-u') - r\alpha \cos (A+B'+u+u') - r'\alpha' \cos (A'-B') + \\
& + r'\alpha' \cos (A'+B'+2u')] r'd\beta' - [-r\beta \cos (B-A'+u-u') + \\
& + r\beta \cos (B+A'+u+u') + r'\beta' \cos (A'-B') - r'\beta' \cos (A' + \\
& + B'+2u)] r'd\alpha' - [r\alpha \sin (A-B'+u-u') + r\alpha \sin (A+B' + \\
& + u+u') - r'\alpha' \sin (A'-B') - r'\alpha' \sin (A'+B'+2u')] r'\beta' dB' - \\
& - [-r\beta \sin (B-A'+u-u') - r\beta \sin (B+A'+u+u') - \\
& - r'\beta' \sin (A'-B') + r'\beta' \sin (A'+B'+2u')] r'\alpha' dA' + \\
& + [2\alpha r\beta \sin (A-B) + r'\beta\alpha' \sin (B-A'+u-u') - \\
& - \beta r'\alpha' \sin (A'+B+u+u') - r'\alpha\beta' \sin (A-B'+u-u') + \\
& + r'\alpha\beta' \sin (A+B'+u+u')] r du - [-2r'\alpha'\beta' \sin (A'-B') + \\
& + r\alpha\beta' \sin (A-B'+u-u') + r\alpha\beta' \sin (A+B'+u+u') - \\
& - r\alpha'\beta \sin (B-A'+u-u') - \alpha'r\beta \sin (B+A'+u+u')] r' du';
\end{aligned}
\tag{12}$$

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$$\begin{aligned}
2\rho^2 d\delta_0 = & \operatorname{tg} \delta_0 [-r\beta^2 + r\beta^2 \cos 2(B+u) + r'\beta\beta' \cos (B-B'+u-u') - \\
& - r'\beta\beta' \cos (B+B'+u+u') - r\alpha^2 + r\alpha^2 \cos 2(A+u) + \\
& + r'\alpha\alpha' \cos (A-A'+u-u') - r'\alpha\alpha' \cos (A'+A+u+u')] dr + \\
& + 2\gamma \sin (C+u) \cos \delta_0 dr + 2r \sin (C+u) \cos \delta_0 d\gamma + \\
& + 2r\gamma \cos \delta_0 \cos (C+u) dC + r \operatorname{tg} \delta_0 [-r\alpha + r\alpha \cos 2(A+u) + \\
& + r'\alpha' \cos (A-A'+u-u') - r'\alpha' \cos (A'+A+u+u')] du + \\
& + r\alpha \operatorname{tg} \delta_0 [-r\alpha \sin 2(A+u) + r'\alpha' \sin (A+A'+u+u') - \\
& - r'\alpha' \sin (A-A'+u-u')] dA + r \operatorname{tg} \delta_0 [-r\beta + r\beta \cos 2(B+u) + \\
& + r'\beta' \cos (B-B'+u-u') - r'\beta' \cos (B+B'+u+u')] d\beta + \\
& + r\beta \operatorname{tg} \delta_0 [-r\beta \sin 2(B+u) - r'\beta' \sin (B-B'+u-u') + \\
& + r'\beta' \sin (B+B'+u+u')] dB + \operatorname{tg} \delta_0 [-r'\beta'^2 + r'\beta'^2 \cos 2(B'+u') + \\
& + r\beta\beta' \cos (B-B'+u-u') - r\beta\beta' \cos (B+B'+u+u') - r'\alpha'^2 + \\
& + r'\alpha'^2 \cos 2(A'+u') + r\alpha\alpha' \cos (A-A'+u-u') - r\alpha\alpha' \cos (A+A' + \\
& + u+u')] dr' - 2\gamma' \sin (C'+u') \cos \delta_0 dr' - 2r' \sin (C'+u') \cos \delta_0 d\gamma' - \\
& - 2r'\gamma' \cos \delta_0 \cos (C'+u') dC' + r' \operatorname{tg} \delta_0 [r\alpha \cos (A-A'+u-u') - \\
& - r\alpha \cos (A+A'+u+u') - r'\alpha' + r'\alpha' \cos 2(A'+u')] d\alpha' +
\end{aligned}
\tag{13}$$

$$\begin{aligned}
& +r'\alpha' \operatorname{tg} \delta_0 [r\alpha \sin(A-A'+u-u') + r\alpha \sin(A+A'+u+u') - \\
& -r'\alpha' \sin 2(A'+u')] dA' + r' \operatorname{tg} \delta_0 [r\beta \cos(B-B'+u-u') - r\beta \cos(B+B'+ \\
& +u+u') - r'\beta' + r'\beta' \cos 2(B'+u')] d\beta' + r'\beta' \operatorname{tg} \delta_0 [r\beta \sin(B-B'+ \\
& +u-u') + r\beta \sin(B+B'+u+u') - r'\beta' \sin 2(B'+u')] dB' + \\
& +r \{ \operatorname{tg} \delta_0 [-r\alpha^2 \sin 2(A+u) + r'\alpha\alpha' \sin(A+A'+u+u') - \\
& -r'\alpha\alpha' \sin(A-A'+u-u') - r\beta^2 \sin 2(B+u) - r'\beta\beta' \sin(B-B'+ \\
& +u-u') + r'\beta\beta' \sin(B+B'+u+u')] + 2\gamma \cos(C+u) \cos \delta_0 \} du + \\
& +r' \{ \operatorname{tg} \delta_0 [r\alpha\alpha' \sin(A-A'+u-u') + r\alpha\alpha' \sin(A+A'+u+u') - \\
& -r'\alpha'^2 \sin 2(A'+u') + r\beta\beta' \sin(B-B'+u-u') + r\beta\beta' \sin(B+B'+u+u') - \\
& -r'\beta'^2 \sin 2(B'+u')] - 2\gamma' \cos(C'+u') \cos \delta_0 \} du'.
\end{aligned} \tag{13}$$

Using the well-known relations [9]

$$\begin{aligned}
\alpha \sin A &= \cos \Omega, \\
\alpha \cos A &= -\sin \Omega \cos i,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\beta \sin B &= \sin \Omega \cos \varepsilon, \\
\beta \cos B &= \cos \Omega \cos i \cos \varepsilon - \sin i \sin \varepsilon,
\end{aligned} \tag{15}$$

$$\begin{aligned}
\gamma \sin C &= \sin \Omega \sin \varepsilon, \\
\gamma \cos C &= \cos \Omega \cos i \sin \varepsilon + \sin i \cos \varepsilon,
\end{aligned} \tag{16}$$

it is easy to obtain expressions for α , β , γ , A , B , C and their differentials:

$$\begin{aligned}
\alpha &= \sqrt{\cos^2 \Omega + \sin^2 \Omega \cos^2 i}, \\
\beta &= \sqrt{\sin^2 \Omega \cos^2 \varepsilon + \cos^2 \Omega \cos^2 i \cos^2 \varepsilon + \sin^2 i \sin^2 \varepsilon - \frac{1}{2} \cos \Omega \sin 2i \sin 2\varepsilon}, \\
\gamma &= \sqrt{\sin^2 \Omega \sin^2 \varepsilon + \cos^2 \Omega \cos^2 i \sin^2 \varepsilon + \sin^2 i \cos^2 \varepsilon + \frac{1}{2} \cos \Omega \sin 2i \sin 2\varepsilon},
\end{aligned} \tag{17}$$

$$\begin{aligned}
\operatorname{ctg} A &= -\operatorname{ctg} \Omega \sec i, \\
\operatorname{ctg} B &= \operatorname{ctg} \Omega \cos i - \sin i \operatorname{cosec} \Omega \operatorname{tg} \varepsilon, \\
\operatorname{ctg} C &= \operatorname{ctg} \Omega \cos i + \sin i \operatorname{cosec} \Omega \operatorname{ctg} \varepsilon,
\end{aligned} \tag{18}$$

$$\begin{aligned}
d\alpha &= sd\Omega + pdi, \\
d\beta &= kd\Omega + md\varepsilon + ndi, \\
d\gamma &= qd\Omega + td\varepsilon + wdi, \\
dA &= Sd\Omega + Pdi, \\
dB &= Kd\Omega + Md\varepsilon + Ndi, \\
dC &= Qd\Omega + Td\varepsilon + Wdi,
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
s &= \frac{\sin 2\Omega (\cos^2 i - 1)}{2\alpha}, \\
p &= -\frac{\sin^2 \Omega \sin 2i}{2\alpha},
\end{aligned} \tag{20}$$

$$\begin{aligned}
k &= \frac{\sin 2\Omega \cos^2 \varepsilon - \sin 2\Omega \cos^2 i \cos^2 \varepsilon + \frac{1}{2} \sin \Omega \sin 2i \sin 2\varepsilon}{2\beta}, \\
m &= \frac{\sin^2 i \sin 2\varepsilon - \sin^2 \Omega \sin 2\varepsilon - \cos^2 \Omega \cos^2 i \sin 2\varepsilon - \cos \Omega \sin 2i \cos 2\varepsilon}{2\beta},
\end{aligned} \tag{21}$$

$$\begin{aligned}
n &= \frac{\sin^2 \varepsilon \sin 2i - \cos^2 \Omega \cos^2 \varepsilon \sin 2i - \cos \Omega \sin 2\varepsilon \cos 2i}{2\beta}, \\
q &= \frac{\sin 2\Omega \sin^2 \varepsilon - \cos^2 i \sin^2 \varepsilon \sin 2\Omega - \frac{1}{2} \sin \Omega \sin 2i \sin 2\varepsilon}{2\gamma}, \\
t &= \frac{\sin^2 \Omega \sin 2\varepsilon + \cos^2 \Omega \cos^2 i \sin 2\varepsilon - \sin^2 i \sin 2\varepsilon + \cos \Omega \sin 2i \cos 2\varepsilon}{2\gamma}, \\
w &= \frac{\cos^2 \varepsilon \sin 2i - \cos^2 \Omega \sin^2 \varepsilon \sin 2i + \cos \Omega \cos 2i \sin 2\varepsilon}{2\gamma}, \\
S &= \frac{\cos i}{\alpha^2}, \\
P &= -\frac{\sin i \sin 2\Omega}{2\alpha^2},
\end{aligned} \tag{22}$$

$$\begin{aligned}
K &= \frac{\cos i \cos^2 \varepsilon - \frac{1}{2} \sin i \cos \Omega \sin 2\varepsilon}{\beta^2}, \\
M &= \frac{\sin i \sin \Omega}{\beta^2}, \\
N &= \frac{\sin 2\Omega \sin i \cos^2 \varepsilon + \cos i \sin \Omega \sin 2\varepsilon}{2\beta^2},
\end{aligned}$$

$$\begin{aligned}
Q &= \frac{\cos i \sin^2 \varepsilon + \frac{1}{2} \sin i \cos \Omega \sin 2\varepsilon}{\gamma^2}, \\
T &= \frac{\sin i \sin \Omega}{\gamma^2}, \\
W &= \frac{\sin i \sin \Omega \sin^2 \varepsilon - \frac{1}{2} \cos i \sin \Omega \sin 2\varepsilon}{\gamma^2}.
\end{aligned}
\tag{11}$$

We write similar relations for the earth:

$$\begin{aligned}
d\alpha' &= s'd\Omega' + p'di', \\
d\beta' &= k'd\Omega' + m'd\varepsilon + n'di',
\end{aligned}
\tag{23}$$

$$\begin{aligned}
d\gamma' &= q'd\Omega' + t'd\varepsilon + w'di', \\
dA' &= S'd\Omega' + P'di', \\
dB' &= K'd\Omega' + M'd\varepsilon + N'di', \\
dC' &= Q'd\Omega' + T'd\varepsilon + W'di'.
\end{aligned}
\tag{24}$$

Now Eqs. (12) and (13) can be transformed so that the corrections to the elements of the orbits of the planet and the earth are separated out. At the present stage one can do this only for those elements which characterize the position of the orbits of the planet and the earth in space:

$$\begin{aligned}
&\frac{2\rho^2}{rr'} \cos^2 \delta_0 d\alpha_0 = \cos (A-B'+u-u') X_1 + \cos (A+B'+u+u') X_2 + \\
&\quad + \cos (B-A'+u-u') X_3 + \cos (B+A'+u+u') X_4 + \\
&+ \frac{r}{r'} \cos (A-B) X_5 + \frac{r}{r'} \cos (A+B+2u) X_6 + \frac{r}{r'} \sin (A-B) X_7 + \\
&+ \frac{r}{r'} \sin (A+B+2u) X_8 + \sin (B-A'+u-u') X_9 + \sin (B+A'+ \\
&\quad + u+u') X_{10} + \sin (A-B'+u-u') X_{11} + \sin (A+B'+u+ \\
&\quad + u') X_{12} + \frac{r'}{r} \cos (A'-B') X'_1 + \frac{r'}{r} \cos (A'+B'+2u') X'_2 + \\
&\quad + \frac{r'}{r} \sin (A'-B') X'_3 + \frac{r'}{r} \sin (A'+B'+2u') X'_4; \\
&\operatorname{ctg} \delta_0 \frac{2\rho^2}{rr'} d\delta_0 = Y_0 + \cos (B-B'+u-u') Y_1 + \cos (B+B'+ \\
&\quad + u+u') Y_2 + \frac{r}{r'} \cos 2(A+u) Y_3 + \frac{r}{r'} \cos 2(B+u) Y_4 + \\
&\quad + \cos (A-A'+u-u') Y_5 + \cos (A+A'+u+u') Y_6 + \\
&+ \frac{\cos^2 \delta_0}{\sin \delta_0 r'} \sin (C+u) Y_7 + \frac{\cos^2 \delta_0}{\sin \delta_0 r'} \cos (C+u) Y_8 + \frac{r}{r'} \sin 2(A+ \\
&\quad + u) Y_9 + \sin (A+A'+u+u') Y_{10} + \sin (A-A'+u-u') Y_{11} + \\
&+ \frac{r}{r'} \sin 2(B+u) Y_{12} + \sin (B-B'+u-u') Y_{13} + \sin (B+B'+u+u') Y_{14} +
\end{aligned}
\tag{25}$$

$$\begin{aligned}
& + \frac{r'}{r} \cos 2(B' + u') Y'_1 + \frac{r'}{r} \cos 2(A' + u') Y'_2 + \frac{\cos^2 \delta_0}{\sin \delta_0 r} \sin (C' + u') Y'_3 + \\
& + \frac{\cos^2 \delta_0}{\sin \delta_0 r} \cos (C' + u') Y'_4 + \frac{r'}{r} \sin 2(A' + u') Y'_5 + \frac{r'}{r} \sin 2(B' + u') Y'_6.
\end{aligned} \tag{26}$$

The quantities X_i and Y_i in Eqs. (25) and (26) have the following values:

$$\begin{aligned}
X_1 &= \alpha \beta' \left(\frac{dr}{r} - \frac{dr'}{r'} \right) + \beta' (sd\Omega + pdi) - \alpha (k'd\Omega' + m'd\varepsilon + n'di'), \\
X_2 &= -X_1, \\
X_3 &= -\beta \alpha' \left(\frac{dr}{r} - \frac{dr'}{r'} \right) - \alpha' (kd\Omega + mde + ndi) + \beta (s'd\Omega' + p'di'), \\
X_4 &= -X_3, \\
X_5 &= (k\alpha - s\beta) d\Omega + (n\alpha - p\beta) di + mde, \\
X_6 &= -X_5, \\
X_7 &= \alpha \beta' [(K + S - 2) d\Omega + (P + N) di + Mde + 2dl], \\
X_8 &= \alpha \beta' [(K - S) d\Omega + (N - P) di + Mde], \\
X_9 &= \alpha' \beta' [(K - 1) d\Omega + Mde + Ndi + (S' - 1) d\Omega' + P'di' + dl + dl'], \\
X_{10} &= -\alpha' \beta' [(K - 1) d\Omega + Mde + Ndi - (S' - 1) d\Omega' - P'di' + dl - dl'], \\
X_{11} &= -\alpha \beta' [(S - 1) d\Omega + Pdi + (K' - 1) d\Omega' + M'd\varepsilon + N'di' + dl + dl'], \\
X_{12} &= \alpha \beta' [(S - 1) d\Omega + Pdi - (K' - 1) d\Omega' - M'd\varepsilon - N'di' + dl - dl'], \\
X'_1 &= (\alpha' k' - \beta' s') d\Omega' + (\alpha' n' - \beta' p') di' + \alpha' m'd\varepsilon, \\
X'_2 &= -X'_1, \\
X'_3 &= \alpha' \beta' [(K' + S' - 2) d\Omega' + (N' + P') di' + M'd\varepsilon + 2dl'], \\
X'_4 &= \alpha' \beta' [(-S' + K') d\Omega' + (-P' + N') di' + M'd\varepsilon], \\
Y_0 &= -(\alpha^2 + \beta^2) \frac{dr}{r} + \frac{r}{r'} [(s\alpha + \beta k) d\Omega + (n\beta + p\alpha) di + m\beta d\varepsilon] + \\
& + (\alpha'^2 + \beta'^2) \frac{dr'}{r'} + [(\alpha' s' + \beta' k') d\Omega' + (\alpha' p' + n' \beta') di' + m' \beta' d\varepsilon] \frac{r'}{r}, \\
Y_1 &= \beta \beta' \left(\frac{dr}{r} + \frac{dr'}{r'} \right) + \beta' (kd\Omega + mde + ndi) + \beta (k'd\Omega' + m'd\varepsilon + n'di'), \\
Y_2 &= -Y_1, \\
Y_3 &= \alpha^2 \frac{dr}{r} + \alpha (sd\Omega + pdi), \\
Y_4 &= \beta^2 \frac{dr}{r} + \beta (kd\Omega + mde + ndi), \\
Y_5 &= \alpha \alpha' \left(\frac{dr}{r} + \frac{dr'}{r'} \right) + \alpha' (sd\Omega + pdi) + \alpha (s'd\Omega' + p'di'), \\
Y_6 &= -Y_5, \\
Y_7 &= 2 \left(\gamma \frac{dr}{r} + qd\Omega + td\varepsilon + wdi \right),
\end{aligned} \tag{27}$$

(28)

$$\begin{aligned}
Y_8 &= 2\gamma[(Q-1)d\Omega + Td\varepsilon + Wdi + d\eta], \\
Y_9 &= -\alpha^2[(S-1)d\Omega + Pdi + d\eta], \\
Y_{10} &= \alpha\alpha'[(S-1)d\Omega + Pdi + (S'-1)d\Omega' + P'di' + dl + dl'], \\
Y_{11} &= \alpha\alpha'[-(S-1)d\Omega - Pdi + (S'-1)d\Omega' + P'di' - dl + dl'], \\
Y_{12} &= -\beta^2[(K-1)d\Omega + Mdi + Ndi + d\eta], \\
Y_{13} &= \beta\beta'[-(K-1)d\Omega - Mdi - Ndi + (K'-1)d\Omega' + \\
&\quad + M'd\varepsilon + N'di' - dl + dl'], \\
Y_{14} &= \beta\beta'[(K-1)d\Omega + Mdi + Ndi + (K'-1)d\Omega' + \\
&\quad + M'd\varepsilon + N'di' + dl + dl'], \\
Y_{1'} &= \beta'^2 \frac{dr'}{r'} + \beta'(k'd\Omega' + m'd\varepsilon + n'di'), \\
Y_{2'} &= \alpha'^2 \frac{dr'}{r'} + \alpha'(s'd\Omega' + p'di'), \\
Y_{3'} &= -2\left(\gamma' \frac{dr'}{r'} + q'd\Omega' + t'd\varepsilon + w'di'\right), \\
Y_{4'} &= -2\gamma'[(Q'-1)d\Omega' + T'd\varepsilon + W'di' + dl'], \\
Y_{5'} &= -\alpha'^2[(S'-1)d\Omega' + P'di' + d\eta], \\
Y_{6'} &= -\beta'^2[(K'-1)d\Omega' + M'd\varepsilon + N'di' + d\eta].
\end{aligned}$$

Assuming in Eqs. (25) and (26)

$$u = l - \Omega, \quad u' = l' - \Omega',$$

we obtain:

$$\begin{aligned}
\frac{2\rho^2}{rr'} \cos^2 \delta_0 d\alpha_0 &= \Pi_1 \cos(l-l') + \Pi_2 \sin(l-l') + \Pi_3 \cos(l+l') + \\
&+ \Pi_4 \sin(l+l') + \frac{r}{r'} \Pi_5 \cos 2l + \frac{r}{r'} \Pi_6 \sin 2l + \frac{r'}{r} \Pi_7 \cos 2l' + \\
&+ \frac{r'}{r} \Pi_8 \sin 2l' + \frac{r}{r'} \Pi_9 + \frac{r'}{r} \Pi_{10};
\end{aligned} \tag{29}$$

$$\begin{aligned}
\frac{2\rho^2}{rr'} \operatorname{ctg} \delta_0 d\delta_0 &= T_1 \cos(l-l') + T_2 \cos(l+l') + T_3 \sin(l-l') + \\
&+ T_4 \sin(l+l') + \frac{r}{r'} T_5 \cos 2l + T_6 \frac{r}{r'} \sin 2l + T_7 \frac{\cos^2 \delta_0}{\sin \delta_0} \frac{1}{r'} \cos l + \\
&+ T_8 \frac{\cos^2 \delta_0}{\sin \delta_0} \frac{1}{r'} \sin l + T_9 \frac{r'}{r} \cos 2l' + T_{10} \frac{r'}{r} \sin 2l' + \\
&+ T_{11} \frac{1}{r} \frac{\cos^2 \delta_0}{\sin \delta_0} \cos l' + T_{12} \frac{1}{r} \frac{\cos^2 \delta_0}{\sin \delta_0} \sin l' + Y_0.
\end{aligned} \tag{30}$$

For compactness in form the following notations are introduced:

$$\begin{aligned}\cos(A-B'-\Omega+\Omega') &= (1), & \cos(A+B'-\Omega-\Omega') &= (5), \\ \cos(B-A'-\Omega+\Omega') &= (2), & \cos(B+A'-\Omega-\Omega') &= (6), \\ \sin(B-A'-\Omega+\Omega') &= (3), & \sin(B+A'-\Omega-\Omega') &= (7), \\ \sin(A-B'-\Omega+\Omega') &= (4), & \sin(A+B'-\Omega-\Omega') &= (8),\end{aligned}$$

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$$\begin{aligned}\cos(A+B-2\Omega) &= (9), & \cos 2(B-\Omega) &= (27), \\ \sin(A+B-2\Omega) &= (10), & \sin 2(B-\Omega) &= (28), \\ \cos(A'+B'-2\Omega') &= (11), & \sin(C-\Omega) &= (29), \\ \sin(A'+B'-2\Omega') &= (12), & \cos(C-\Omega) &= (30), \\ \cos(A-B) &= (13), & \cos 2(B'-\Omega') &= (31), \\ \sin(A-B) &= (14), & \sin 2(B'-\Omega') &= (32), \\ \cos(A'-B') &= (15), & \cos 2(A'-\Omega') &= (33), \\ \sin(A'-B') &= (16), & \sin 2(A'-\Omega') &= (34), \\ \cos(B-B'-\Omega+\Omega') &= (17), & \sin(C'-\Omega') &= (35), \\ \cos(B+B'-\Omega-\Omega') &= (18), & \cos(C'-\Omega') &= (36), \\ \sin(B-B'-\Omega+\Omega') &= (19), & \sin(A-\Omega) &= (37), \\ \sin(B+B'-\Omega-\Omega') &= (20), & \cos(A-\Omega) &= (38), \\ \cos(A-A'-\Omega+\Omega') &= (21), & \sin(A'-\Omega') &= (39), \\ \cos(A+A'-\Omega-\Omega') &= (22), & \cos(A'-\Omega') &= (40), \\ \sin(A-A'-\Omega+\Omega') &= (23), & \sin(B-\Omega) &= (41), \\ \sin(A+A'-\Omega-\Omega') &= (24), & \cos(B-\Omega) &= (42), \\ \cos 2(A-\Omega) &= (25), & \sin(B'-\Omega') &= (43), \\ \sin 2(A-\Omega) &= (26), & \cos(B'-\Omega') &= (44);\end{aligned}\tag{31}$$

$$\begin{aligned}\Pi_1 &= (1)X_1 + (2)X_3 + (3)X_9 + (4)X_{11} = \\ &= \sum_{i=1}^5 F_i + F_6 \left(\frac{dr}{r} - \frac{dr'}{r'} \right) + F_7 (dl + dl'), \\ \Pi_2 &= -(4)X_1 - (3)X_3 + (2)X_9 + (1)X_{11} = \\ &= \sum_{i=8}^{12} F_i + F_{13} \left(\frac{dr}{r} - \frac{dr'}{r'} \right) + F_{14} (dl + dl'), \\ \Pi_3 &= (5)X_2 + (6)X_4 + (7)X_{10} + (8)X_{12} = \\ &= \sum_{i=15}^{19} F_i + F_{20} \left(\frac{dr}{r} - \frac{dr'}{r'} \right) + F_{21} (dl - dl'), \\ \Pi_4 &= -(8)X_2 - (7)X_4 + (6)X_{10} + (5)X_{12} = \\ &= \sum_{i=22}^{26} F_i + F_{27} \left(\frac{dr}{r} - \frac{dr'}{r'} \right) + F_{28} (dl - dl'),\end{aligned}\tag{32}$$

$$\begin{aligned}
\Pi_5 &= (9)X_6 + (10)X_8 = \sum_{i=29}^{31} F_i, \\
\Pi_6 &= -(10)X_6 + (9)X_8 = \sum_{i=32}^{34} F_i, \\
\Pi_7 &= (11)X'_2 + (12)X'_4 = \sum_{i=35}^{37} F_i, \\
\Pi_8 &= -(12)X'_2 + (11)X'_4 = \sum_{i=38}^{40} F_i, \\
\Pi_9 &= (13)X_5 + (14)X_7 = \sum_{i=41}^{43} F_i + F_{44}dl, \\
\Pi_{10} &= (15)X'_1 + (16)X'_3 = \sum_{i=45}^{47} F_i + F_{48}dl'; \\
T_1 &= (17)Y_1 + (21)Y_5 + (23)Y_{11} + (19)Y_{13} = \\
&= \sum_{i=49}^{53} F_i + F_{54} \left(\frac{dr}{r} + \frac{dr'}{r'} \right) + F_{55}(-dl + dl'), \\
T_2 &= (18)Y_2 + (22)Y_6 + (24)Y_{10} + (20)Y_{14} = \\
&= \sum_{i=56}^{60} F_i + F_{61} \left(\frac{dr}{r} + \frac{dr'}{r'} \right) + F_{62}(dl + dl'), \\
T_3 &= -(19)Y_1 - (23)Y_5 + (21)Y_{11} + (17)Y_{13} = \\
&= \sum_{i=63}^{67} F_i + F_{68} \left(\frac{dr}{r} + \frac{dr'}{r'} \right) + F_{69}(-dl + dl'), \\
T_4 &= -(20)Y_2 - (24)Y_6 + (22)Y_{10} + (18)Y_{14} = \\
&= \sum_{i=70}^{74} F_i + F_{75} \left(\frac{dr}{r} + \frac{dr'}{r'} \right) + F_{76}(dl + dl'), \\
T_5 &= (25)Y_3 + (27)Y_4 + (26)Y_9 + (28)Y_{12} = \sum_{i=77}^{79} F_i + F_{80} \frac{dr}{r} + F_{81}dl, \\
T_6 &= -(26)Y_3 - (28)Y_4 + (25)Y_9 + (27)Y_{12} = \sum_{i=82}^{84} F_i + F_{85} \frac{dr}{r} + F_{86}dl, \\
T_7 &= (29)Y_7 + (30)Y_8 = \sum_{i=87}^{89} F_i + F_{90} \frac{dr}{r} + F_{91}dl, \\
T_8 &= (30)Y_7 - (29)Y_8 = \sum_{i=92}^{94} F_i + F_{95} \frac{dr}{r} + F_{96}dl,
\end{aligned} \tag{33}$$

$$T_9 = (31) Y'_1 + (33) Y'_2 + (34) Y'_5 + (32) Y'_6 = \sum_{i=97}^{99} F_i + F_{100} \frac{dr'}{r'} + F_{101} dl',$$

$$T_{10} = -(32) Y'_1 - (34) Y'_2 + (33) Y'_5 + (31) Y'_6 = \sum_{i=102}^{104} F_i + F_{105} \frac{dr'}{r'} + F_{106} dl',$$

$$T_{11} = (35) Y'_2 + (36) Y'_4 = \sum_{i=107}^{109} F_i + F_{110} \frac{dr'}{r'} + F_{111} dl',$$

$$T_{12} = (36) Y'_3 - (35) Y'_4 = \sum_{i=112}^{114} F_i + F_{115} \frac{dr'}{r'} + F_{116} dl';$$

$$\begin{aligned} F_1 &= [(1) \beta' s - (2) \alpha' k + (3) \alpha' \beta (K-1) - (4) \alpha \beta' (S-1)] d\Omega, \\ F_2 &= [(1) \beta' p - (2) \alpha' n + (3) \alpha' \beta N - (4) \alpha \beta' P] di, \\ F_3 &= [-(1) \alpha k' + (2) \beta s' + (3) \alpha' \beta (S'-1) - (4) \alpha \beta' (K'-1)] d\Omega', \\ F_4 &= [-(1) \alpha m' - (2) \alpha' m + (3) \alpha' \beta M - (4) \alpha \beta' M'] d\epsilon, \\ F_5 &= [-(1) \alpha n' + (2) \beta p' + (3) \alpha' \beta P' - (4) \alpha \beta' N'] di', \\ F_6 &= [(1) \alpha \beta' - (2) \beta \alpha'], \\ F_7 &= [(3) \alpha' \beta - (4) \alpha \beta'], \\ F_8 &= [-(4) \beta' s + (3) \alpha' k + (2) \alpha' \beta (K-1) - (1) \alpha \beta' (S-1)] d\Omega, \\ F_9 &= [-(4) \beta' p + (3) \alpha' n + (2) \alpha' \beta N - (1) \alpha \beta' P] di, \\ F_{10} &= [(4) \alpha k' - (3) \beta s' + (2) \alpha' \beta (S'-1) - (1) \alpha \beta' (K'-1)] d\Omega', \\ F_{11} &= [(4) \alpha m' + (3) \alpha' m + (2) \alpha' \beta M - (1) \alpha \beta' M'] d\epsilon, \\ F_{12} &= [(4) \alpha n' - (3) \beta p' + (2) \alpha' \beta P' - (1) \alpha \beta' N'] di, \\ F_{13} &= -(4) \alpha \beta' + (3) \beta \alpha', \\ F_{14} &= (2) \alpha' \beta - (1) \alpha \beta', \\ F_{15} &= [-(5) \beta' s + (6) \alpha' k - (7) \alpha' \beta (K-1) + (8) \alpha \beta' (S-1)] d\Omega, \\ F_{16} &= [-(5) \beta' p + (6) \alpha' n - (7) \alpha' \beta N + (8) \alpha \beta' P] di, \\ F_{17} &= [(5) \alpha k' - (6) \beta s' + (7) \alpha' \beta (S'-1) - (8) \alpha \beta' (K'-1)] d\Omega', \\ F_{18} &= [(5) \alpha m' + (6) \alpha' m - (7) \alpha' \beta M - (8) \alpha \beta' M'] d\epsilon, \\ F_{19} &= [(5) \alpha n' - (6) \beta p' + (7) \alpha' \beta P' - (8) \alpha \beta' N'] di', \\ F_{20} &= -(5) \alpha \beta' + (6) \beta \alpha', \\ F_{21} &= -(7) \alpha' \beta + (8) \alpha \beta', \\ F_{22} &= [(8) \beta' s - (7) \alpha' k - (6) \alpha' \beta (K-1) + (5) \alpha \beta' (S-1)] d\Omega, \\ F_{23} &= [(8) \beta' p - (7) \alpha' n - (6) \alpha' \beta N + (5) \alpha \beta' P] di, \\ F_{24} &= [-(8) \alpha k' + (7) \beta s' + (6) \alpha' \beta (S'-1) - (5) \alpha \beta' (K'-1)] d\Omega', \\ F_{25} &= [-(8) \alpha m' - (7) \alpha' m - (6) \alpha' \beta M - (5) \alpha \beta' M'] d\epsilon, \\ F_{26} &= [-(8) \alpha n' + (7) \beta p' + (6) \alpha' \beta P' - (5) \alpha \beta' N'] di', \\ F_{27} &= (8) \alpha \beta' - (7) \beta \alpha', \end{aligned} \tag{34}$$

$$\begin{aligned}
F_{28} &= -(6)\alpha'\beta + (5)\alpha\beta', \\
F_{29} &= [-(9)(k\alpha - s\beta) + (10)\alpha\beta(K - S)]d\Omega, \\
F_{30} &= [-(9)(n\alpha - p\beta) + (10)\alpha\beta(N - P)]di, \\
F_{31} &= [-(9)m\alpha + (10)\alpha\beta M]d\varepsilon, \\
F_{32} &= [(10)(k\alpha - s\beta) + (9)\alpha\beta(K - S)]d\Omega, \\
F_{33} &= [(10)(n\alpha - p\beta) + (9)\alpha\beta(N - P)]di, \\
F_{34} &= [(10)m\alpha + (9)\alpha\beta M]d\varepsilon, \\
F_{35} &= [-(11)(\alpha'k' - \beta's') + (12)\alpha'\beta'(K' - S')]d\Omega',
\end{aligned}$$

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$$\begin{aligned}
F_{36} &= [-(11)(\alpha'n' - \beta'p') + (12)\alpha'\beta'(N' - P')]di', \\
F_{37} &= [-(11)\alpha'm' + (12)\alpha'\beta'M']d\varepsilon, \\
F_{38} &= [(12)(\alpha'k' - \beta's') + (11)\alpha'\beta'(K' - S')]d\Omega', \\
F_{39} &= [(12)(\alpha'n' - \beta'p') + (11)\alpha'\beta'(N' - P')]di', \\
F_{40} &= [(12)\alpha'm' + (11)\alpha'\beta'M']d\varepsilon, \\
F_{41} &= [(13)(k\alpha - s\beta) + (14)\alpha\beta(K + S - 2)]d\Omega, \\
F_{42} &= [(13)(n\alpha - p\beta) + (14)\alpha\beta(P + N)]di, \\
F_{43} &= [(13)m\alpha + (14)\alpha\beta M]d\varepsilon, \\
F_{44} &= 2(14)\alpha\beta, \\
F_{45} &= [(15)(\alpha'k' - \beta's') + (16)\alpha'\beta'(K' + S' - 2)]d\Omega', \\
F_{46} &= [(15)(\alpha'n' - \beta'p') + (16)\alpha'\beta'(N' + P')]di', \\
F_{47} &= [(15)\alpha'm' + (16)\alpha'\beta'M']d\varepsilon, \\
F_{48} &= 2(16)\alpha'\beta', \\
F_{49} &= [(17)\beta'k + (21)\alpha's - (23)\alpha\alpha'(S - 1) - (19)\beta\beta'(K - 1)]d\Omega, \\
F_{50} &= [(17)(\beta'm + \beta m') + (19)\beta\beta'(M' - M)]d\varepsilon, \\
F_{51} &= [(17)\beta'n + (21)\alpha'p - (23)\alpha\alpha'P - (19)\beta\beta'N]di, \\
F_{52} &= [(17)\beta'k' + (21)\alpha s' + (23)\alpha\alpha'(S' - 1) + (19)\beta\beta'(K' - 1)]d\Omega', \\
F_{53} &= [(17)\beta n' + (21)\alpha p' + (23)\alpha\alpha'P' + (19)\beta\beta'N']di', \\
F_{54} &= \beta\beta'(17) + (21)\alpha\alpha', \\
F_{55} &= (23)\alpha\alpha' + (19)\beta\beta', \\
F_{56} &= [-(18)\beta'k - (22)\alpha's + (24)\alpha\alpha'(S - 1) + (20)\beta\beta'(K - 1)]d\Omega, \\
F_{57} &= [-(18)(\beta'm + \beta m') + (20)\beta\beta'(M + M')]d\varepsilon, \\
F_{58} &= [-(18)\beta'n - (22)\alpha'p + (24)\alpha\alpha'P + (20)\beta\beta'N]di, \\
F_{59} &= [-(18)\beta'k' - (22)\alpha s' + (24)\alpha\alpha'(S' - 1) + (20)\beta\beta'(K' - 1)]d\Omega', \\
F_{60} &= [-(18)\beta n' - (22)\alpha p' + (24)\alpha\alpha'P' + (20)\beta\beta'N']di', \\
F_{61} &= -(18)\beta\beta' - (22)\alpha\alpha', \\
F_{62} &= (24)\alpha\alpha' + (20)\beta\beta', \\
F_{63} &= [-(19)\beta'k - (23)\alpha's - (21)\alpha\alpha'(S - 1) - (17)\beta\beta'(K - 1)]d\Omega, \\
F_{64} &= [-(19)(\beta'm + \beta m') - (17)\beta\beta'(M - M')]d\varepsilon, \\
F_{65} &= [-(19)\beta'n - (23)\alpha'p - (21)\alpha\alpha'P - (17)\beta\beta'N]di, \\
F_{66} &= [-(19)\beta'k' - (23)\alpha s' + (21)\alpha\alpha'(S' - 1) + (17)\beta\beta'(K' - 1)]d\Omega', \\
F_{67} &= [-(19)\beta n' - (23)\alpha p' + (21)\alpha\alpha'P' + (17)\beta\beta'N']di', \\
F_{68} &= -(19)\beta\beta' - (23)\alpha\alpha',
\end{aligned}$$

$$\begin{aligned}
F_{69} &= (21) \alpha \alpha' + (17) \beta \beta', \\
F_{70} &= [(20) \beta' k + (24) \alpha' s + (22) \alpha \alpha' (S-1) + (18) \beta \beta' (K-1)] d\Omega, \\
F_{71} &= [(20) (\beta' m + \beta m') + (18) \beta \beta' (M+M')] d\varepsilon, \\
F_{72} &= [(20) \beta' n + (24) \alpha' p + (22) \alpha \alpha' P + (18) \beta \beta' N] di, \\
F_{73} &= [(20) \beta k' + (24) \alpha s' + (22) \alpha \alpha' (S'-1) + (18) \beta \beta' (K'-1)] d\Omega', \\
F_{74} &= [(20) \beta n' + (24) \alpha p' + (22) \alpha \alpha' P' + (18) \beta \beta' N'] di', \\
F_{75} &= (20) \beta \beta' + (24) \alpha \alpha', \\
F_{76} &= (22) \alpha \alpha' + (18) \beta \beta', \\
F_{77} &= [(25) \alpha s + (27) \beta k - (26) \alpha^2 (S-1) - (28) \beta^2 (K-1)] d\Omega \\
F_{78} &= [(27) \beta m - (28) \beta^2 M] d\varepsilon, \\
F_{79} &= [(25) \alpha p + (27) \beta n - (26) \alpha^2 P - (28) \beta^2 N] di, \\
F_{80} &= (25) \alpha^2 + (27) \beta^2, \\
F_{81} &= -(26) \alpha^2 - (28) \beta^2, \\
F_{82} &= [-(26) \alpha s - (28) \beta k - (25) \alpha^2 (S-1) - (27) \beta^2 (K-1)] d\Omega \\
F_{83} &= [-(28) \beta m - (27) \beta^2 M] d\varepsilon, \\
F_{84} &= [-(26) \alpha p - (28) \beta n - (25) \alpha^2 P - (27) \beta^2 N] di, \\
F_{85} &= -(26) \alpha^2 - (28) \beta^2, \\
F_{86} &= -(25) \alpha^2 - (27) \beta^2, \\
F_{87} &= [(29) 2q + (30) 2\gamma (Q-1)] d\Omega, \\
F_{88} &= [(29) 2t + (30) 2\gamma T] d\varepsilon, \\
F_{89} &= [(29) 2w + (30) 2\gamma W] di, \\
F_{90} &= (29) 2\gamma, \\
F_{91} &= (30) 2\gamma, \\
F_{92} &= [(30) 2q - (29) 2\gamma (Q-1)] d\Omega, \\
F_{93} &= [(30) 2t - (29) 2\gamma T] d\varepsilon, \\
F_{94} &= [(30) 2w - (29) 2\gamma W] di, \\
F_{95} &= (30) 2\gamma, \\
F_{96} &= -(29) 2\gamma, \\
F_{97} &= [(31) \beta' m' - (32) \beta'^2 M'] d\varepsilon, \\
F_{98} &= [(31) \beta' k' + (33) \alpha' s' - (34) \alpha'^2 (S'-1) - (32) \beta'^2 (K'-1)] d\Omega' \\
F_{99} &= [(31) \beta' n' + (33) \alpha' p' - (34) \alpha'^2 P' - (32) \beta'^2 N'] di', \\
F_{100} &= (31) \beta'^2 + (33) \alpha'^2, \\
F_{101} &= -(34) \alpha'^2 - (32) \beta'^2, \\
F_{102} &= [-(32) \beta' m' - (31) \beta'^2 M'] d\varepsilon, \\
F_{103} &= [-(32) \beta' k' - (34) \alpha' s' - (33) \alpha'^2 (S'-1) - (31) \beta'^2 (K'-1)] d\Omega' \\
F_{104} &= [-(32) \beta' n' - (34) \alpha' p' - (33) \alpha'^2 P' - (31) \beta'^2 N'] di', \\
F_{105} &= -(32) \beta'^2 - (34) \alpha'^2, \\
F_{106} &= -(33) \alpha'^2 - (31) \beta'^2, \\
F_{107} &= [-(35) 2t' - (36) 2\gamma' T'] d\varepsilon, \\
F_{108} &= [-(35) 2q' - (36) 2\gamma' (Q'-1)] d\Omega', \\
F_{109} &= [-(35) 2w' - (36) 2\gamma' W'] di',
\end{aligned}$$

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$$\begin{aligned}
F_{110} &= -(35)2\gamma', \\
F_{111} &= -(36)2\gamma', \\
F_{112} &= [-(36)2l' + (35)2\gamma'T']d\varepsilon, \\
F_{113} &= [-(36)2q' + (35)2\gamma'(Q'-1)]d\Omega', \\
F_{114} &= [-(36)2\omega' + (35)2\gamma'W']di', \\
F_{115} &= -(36)2\gamma', \\
F_{116} &= (35)2\gamma'.
\end{aligned}$$

The differentials of the longitude $d\mathfrak{l}$ and the radius vector dr can be expressed also as corrections to the elements which characterize the position of an object in orbit:

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$$dr = \Theta_1 \sin l + \Theta_2 \cos l + \Theta_3 \tau \sin l + \Theta_4 \tau \cos l + \Theta_5 r, \quad (35)$$

$$d\mathfrak{l} = \psi_1 + \frac{\psi_2}{r^2} + \frac{\psi_3}{r^2} \tau + \frac{p+r}{r} \psi_4 \sin l + \frac{p+r}{r} \psi_5 \cos l, \quad (36)$$

where

$$\begin{aligned}
\Theta_1 &= a[\operatorname{tg} \varphi \cos (\Omega + \omega) dM_0 - \cos \varphi \sin (\Omega + \omega) d\varphi], \\
\Theta_2 &= -a[\operatorname{tg} \varphi \sin (\Omega + \omega) dM_0 + \cos \varphi \cos (\Omega + \omega) d\varphi], \\
\Theta_3 &= a \operatorname{tg} \varphi \cos (\Omega + \omega) d\mu, \\
\Theta_4 &= -a \operatorname{tg} \varphi \sin (\Omega + \omega) d\mu, \\
\Theta_5 &= -\frac{2}{3\mu} d\mu, \\
\psi_1 &= d\Omega + d\omega, \\
\psi_2 &= a^2 \cos \varphi dM_0, \\
\psi_3 &= a^2 \cos \varphi d\mu, \\
\psi_4 &= \sec \varphi \cos (\Omega + \omega) d\varphi, \\
\psi_5 &= -\sec \varphi \sin (\Omega + \omega) d\varphi, \\
\psi'_0 &= \psi_1 - \psi'_1, \\
\psi_0 &= \psi_1 + \psi'_1, \\
\tau &= t - t_0
\end{aligned}$$

For the Earth dr' and $d\mathfrak{l}'$ have the form of Eqs. (35) and (36), but all the quantities entering into them have primes. Although the form of Eqs. (29) and (30) are simple, they are unsuitable for the analysis and calculations. Therefore, they are subsequently presented in a form suitable for theoretical investigations by means of uncomplicated but rather cumbersome transformations. We now substitute $d\alpha_0$ and $d\delta_0$ into Eq. (1). On the basis of Eqs. (2), (4) and (5)

$$\begin{aligned}\cos \alpha_0 &= \frac{(37) r \alpha}{\rho \cos \delta_0} \cos l + \frac{(38) r \alpha}{\rho \cos \delta_0} \sin l - \frac{(39) r' \alpha'}{\rho \cos \delta_0} \cos l' - \frac{(40) r' \alpha'}{\rho \cos \delta_0} \sin l', \\ \sin \alpha_0 &= \frac{(41) r \beta}{\rho \cos \delta_0} \cos l + \frac{(42) r \beta}{\rho \cos \delta_0} \sin l - \frac{(43) r' \beta'}{\rho \cos \delta_0} \cos l' - \frac{(44) r' \beta'}{\rho \cos \delta_0} \sin l' .\end{aligned}$$

After the necessary transformations of the Eq. (1) we finally have the following form:

$$\begin{aligned}\frac{2\rho^2}{rr'} \cos^2 \delta_0 \Delta \alpha_0 &= \Phi_1 \cos (l-l') + \Phi_2 \sin (l-l') + \Phi_3 \cos (l+ \\ &+ l') + \Phi_4 \sin (l+l') + \Phi_5 \cos 2l + \Phi_6 \sin 2l + \Phi_7 \cos 2l' + \\ &+ \Phi_8 \sin 2l' + \Phi_9 \sin l' + \Phi_{10} \cos l' + \Phi_{11} \sin l + \Phi_{12} \cos l + \Phi_{13} \sin (2l-l') + \\ &+ \Phi_{14} \cos (2l-l') + \Phi_{15} \sin (2l'-l) + \Phi_{16} \cos (2l'-l) + \\ &+ \Phi_{17} \sin (2l+l') + \Phi_{18} \cos (2l+l') + \Phi_{19} \sin (2l'+l) + \\ &+ \Phi_{20} \cos (2l'+l) + \Phi_{21} + \Phi_{22} \cdot \tau + \frac{2\rho^2}{rr'} \cos^2 \delta_0 a + \frac{b(37) r \alpha + c(41) r \beta}{\rho \cos \delta_0} \cos l + \\ &+ \frac{b(38) r \alpha + c(42) r \beta}{\rho \cos \delta_0} \sin l - \frac{b(39) r' \alpha' + c(43) r' \beta'}{\rho \cos \delta_0} \cos l' - \\ &- \frac{b(40) r' \alpha' + c(44) r' \beta'}{\rho \cos \delta_0} \sin l';\end{aligned}\tag{37}$$

$$\begin{aligned}\operatorname{ctg} \delta_0 \frac{2\rho^2}{rr'} \Delta \delta_0 &= \Phi_{23} \cos (l-l') + \Phi_{24} \cos (l+l') + \Phi_{25} \sin (l-l') + \\ &+ \Phi_{26} \sin (l+l') + \Phi_{27} \cos 2l + \Phi_{28} \sin 2l + \\ &+ \Phi_{29} \cos l + \Phi_{30} \sin l + \Phi_{31} \cos 2l' + \Phi_{32} \sin 2l' + \\ &+ \Phi_{33} \cos l' + \Phi_{34} \sin l' + \Phi_{35} \cos (2l-l') + \\ &+ \Phi_{36} \cos (2l+l') + \Phi_{37} \sin (2l-l') + \Phi_{38} \sin (2l+l') + \\ &+ \Phi_{39} \cos (2l'-l) + \Phi_{40} \cos (2l'+l) + \Phi_{41} \sin (2l'-l) + \\ &+ \Phi_{42} \sin (2l'+l) + \Phi_{43} \cos 3l + \Phi_{44} \sin 3l + \Phi_{45} \cos 3l' + \\ &+ \Phi_{46} \sin 3l' + \Phi_{47} + \Phi_{48} \tau + \frac{2\rho^2}{rr'} \operatorname{ctg} \delta_0 a' + \frac{b'(37) r \alpha + c'(41) r \beta}{\rho \cos \delta_0} \cos l + \\ &+ \frac{b'(38) r \alpha + c'(42) r \beta}{\rho \cos \delta_0} \sin l - \frac{b'(39) r' \alpha' + c'(43) r' \beta'}{\rho \cos \delta_0} \cos l' - \\ &- \frac{b'(40) r' \alpha' + c'(44) r' \beta'}{\rho \cos \delta_0} \sin l';\end{aligned}\tag{38}$$

$$\begin{aligned}
 \Phi_1 &= \sum_{i=1}^5 F_i + F_6 (\Theta_5 - \Theta'_5) + F_7 \left[\left(\psi_0 + \frac{\psi_2}{r_2} + \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} + \frac{\psi'_3}{r'^2} \right) \right], \\
 \Phi_2 &= \sum_{i=8}^{12} F_i + F_{13} (\Theta_5 - \Theta'_5) + F_{14} \left[\left(\psi_0 + \frac{\psi_2}{r^2} + \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} + \frac{\psi'_3}{r'^2} \right) \right], \\
 \Phi_3 &= \sum_{i=15}^{19} F_i + F_{20} (\Theta_5 - \Theta'_5) + F_{21} \left[\left(\psi'_0 + \frac{\psi_2}{r^2} - \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} - \frac{\psi'_3}{r'^2} \right) \right], \\
 \Phi_4 &= \sum_{i=22}^{26} F_i + F_{27} (\Theta_5 - \Theta'_5) + F_{28} \left[\left(\psi'_0 + \frac{\psi_2}{r^2} - \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} - \frac{\psi'_3}{r'^2} \right) \right], \\
 \Phi_5 &= \frac{r}{r'} \sum_{i=29}^{31} F_i, \\
 \Phi_6 &= \frac{r}{r'} \sum_{i=32}^{34} F_i, \\
 \Phi_7 &= \frac{r'}{r} \sum_{i=35}^{37} F_i, \\
 \Phi_8 &= \frac{r'}{r} \sum_{i=38}^{40} F_i,
 \end{aligned}$$

$$\begin{aligned}
 \Phi_9 &= \frac{1}{2r} \{ (\Theta_1 + \tau \Theta_3) (F_6 - F_{20}) - (\Theta_2 + \tau \Theta_4) (F_{13} - F_{27}) + \\
 &\quad + (p+r) [\psi_4 (F_7 - F_{21}) - \psi_5 (F_{14} - F_{28})] + 2(p'+r') F_{48} \psi'_4 \}, \\
 \Phi_{10} &= \frac{1}{2r} \{ (\Theta_1 + \tau \Theta_3) (F_{13} + F_{27}) + (\Theta_2 + \tau \Theta_4) (F_6 + F_{20}) + \\
 &\quad + (p+r) [\psi_5 (F_7 + F_{21}) + \psi_4 (F_{14} + F_{28})] + 2(p'+r') F_{48} \psi'_5 \},
 \end{aligned}$$

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$$\begin{aligned}
 \Phi_{11} &= \frac{1}{2r'} \{ -(\Theta'_1 + \tau \Theta'_3) (F_6 - F_{20}) - (\Theta'_2 + \tau \Theta'_4) (F_{13} + F_{27}) + \\
 &\quad + (p'+r') [\psi'_4 (F_7 + F_{21}) - \psi'_5 (F_{14} - F_{28})] + 2(p+r) F_{44} \psi_4 \}, \\
 \Phi_{12} &= \frac{1}{2r'} \{ (\Theta'_1 + \tau \Theta'_3) (F_{13} - F_{27}) - (\Theta'_2 + \tau \Theta'_4) (F_6 + F_{20}) + \\
 &\quad + (p'+r') [\psi'_5 (F_7 - F_{21}) - \psi'_4 (F_{14} - F_{28})] + 2(p+r) F_{44} \psi'_5 \},
 \end{aligned}$$

$$\Phi_{13} = \frac{1}{2r} [F_6 (\Theta_1 + \tau \Theta_3) + F_{13} (\Theta_2 + \tau \Theta_4) + (p+r) (F_7 \psi_4 + F_{14} \psi_5)],$$

$$\Phi_{14} = \frac{1}{2r} [F_6 (\Theta_2 + \tau \Theta_4) - F_{13} (\Theta_1 + \tau \Theta_3) + (p+r) (F_7 \psi_5 - F_{14} \psi_4)],$$

$$\Phi_{15} = \frac{1}{2r'} [-F_6 (\Theta'_1 + \tau \Theta'_3) + F_{13} (\Theta'_2 + \tau \Theta'_4) + (p'+r') (F_7 \psi'_4 - F_{14} \psi'_5)],$$

$$\Phi_{16} = \frac{1}{2r'} [-F_6 (\Theta'_2 + \tau \Theta'_4) - F_{13} (\Theta'_1 + \tau \Theta'_3) + (p'+r') (F_7 \psi'_5 + F_{14} \psi'_4)],$$

$$\begin{aligned}
\psi_{17} &= \frac{1}{2r} [F_{20}(\Theta_1 + \tau\Theta_3) + F_{27}(\Theta_2 + \tau\Theta_4) + (p+r)(F_{21}\psi_4 + F_{28}\psi_5)], \\
\psi_{18} &= \frac{1}{2r} [F_{20}(\Theta_2 + \tau\Theta_4) - F_{27}(\Theta_1 + \tau\Theta_3) + (p+r)(F_{21}\psi_5 - F_{28}\psi_4)], \\
\psi_{19} &= \frac{1}{2r'} [-F_{20}(\Theta'_1 + \tau\Theta'_3) - F_{27}(\Theta'_2 + \tau\Theta'_4) + (p'+r')(F_{21}\psi'_4 + F_{28}\psi'_5)], \\
\psi_{20} &= \frac{1}{2r'} [-F_{20}(\Theta'_2 + \tau\Theta'_4) + F_{27}(\Theta'_1 + \tau\Theta'_3) + (p'+r')(F_{21}\psi'_5 + F_{28}\psi'_4)], \\
\psi_{21} &= \frac{1}{rr'} \left\{ r^2 \left[\sum_{i=41}^{43} F_i + \left(\psi_1 + \frac{\psi_2}{r^2} \right) F_{44} \right] + \right. \\
&\quad \left. + r'^2 \left[\sum_{i=45}^{47} F_i + \left(\psi'_1 + \frac{\psi'_2}{r'^2} \right) F_{48} \right] \right\}, \\
\psi_{22} &= \frac{1}{rr'} (F_{44}\psi_3 + F_{48}\psi'_3), \\
\psi_{23} &= \sum_{i=49}^{53} F_i + F_{54}(\Theta_5 + \Theta'_5) - F_{55} \left[\left(\psi_0 + \frac{\psi_2}{r^2} - \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} - \frac{\psi'_3}{r'^2} \right) \right], \\
\psi_{24} &= \sum_{i=56}^{60} F_i + F_{61}(\Theta_5 + \Theta'_5) + F_{62} \left[\left(\psi_0 + \frac{\psi_2}{r^2} + \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} + \frac{\psi'_3}{r'^2} \right) \right], \\
\psi_{25} &= \sum_{i=63}^{67} F_i + F_{68}(\Theta_5 + \Theta'_5) - F_{69} \left[\left(\psi_0 + \frac{\psi_2}{r^2} - \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} - \frac{\psi'_3}{r'^2} \right) \right], \\
\psi_{26} &= \sum_{i=70}^{74} F_i + F_{75}(\Theta_5 + \Theta'_5) + F_{76} \left[\left(\psi_0 + \frac{\psi_2}{r^2} + \frac{\psi'_2}{r'^2} \right) + \tau \left(\frac{\psi_3}{r^2} + \frac{\psi'_3}{r'^2} \right) \right], \\
\psi_{27} &= \frac{1}{2rr'} \left\{ 2r^2 \left[\sum_{i=77}^{79} F_i + F_{80}\Theta_5 + F_{81} \left(\psi_1 + \frac{\psi_2}{r^2} \right) \right] + \frac{\cos^2 \delta_0}{\sin \delta_0} [F_{90}\Theta_2 - \right. \\
&\quad \left. - F_{95}\Theta_1 + (p+r)(\psi_5 F_{91} - \psi_4 F_{96})] + \tau \left[2\psi_3 F_{81} + \right. \right. \\
&\quad \left. \left. + \frac{\cos^2 \delta_0}{\sin \delta_0} (F_{90}\Theta_4 - F_{95}\Theta_3) \right] \right\}, \\
\psi_{28} &= \frac{1}{2rr'} \left\{ 2r^2 \left[\sum_{i=82}^{84} F_i + F_{85}\Theta_5 + F_{86} \left(\psi_1 + \frac{\psi_2}{r^2} \right) \right] + \frac{\cos^2 \delta_0}{\sin \delta_0} [F_{90}\Theta_1 + \right. \\
&\quad \left. + F_{95}\Theta_2 + (p+r)(\psi_4 F_{91} + \psi_5 F_{96})] + \tau \left[2\psi_3 F_{86} + \right. \right. \\
&\quad \left. \left. + \frac{\cos^2 \delta_0}{\sin \delta_0} (F_{90}\Theta_3 + F_{95}\Theta_4) \right] \right\}, \\
\psi_{29} &= \frac{1}{2r'} \left\{ 2 \frac{\cos^2 \delta_0}{\sin \delta_0} \left[\sum_{i=87}^{89} F_i + F_{90}\Theta_5 + F_{91} \left(\psi_1 + \frac{\psi_2}{r^2} \right) \right] + F_{85}\Theta_1 + \right. \\
&\quad \left. + [F_{80} - 2(\alpha^2 + \beta^2)]\Theta_2 + (p+r)(\psi_5 F_{81} + \psi_4 F_{86}) + (F_{54} + F_{61})\Theta'_2 - \right. \\
&\quad \left. - (F_{68} - F_{75})\Theta'_1 + (p'+r')[\psi'_5 (F_{55} + F_{62}) - \psi'_4 (F_{69} - F_{76})] + \right.
\end{aligned}$$

$$\begin{aligned}
& + \tau \left\{ 2 \frac{\cos^2 \delta_0}{\sin \delta_0} \frac{\psi_3}{r^2} F_{91} + F_{83} \Theta_3 + [F_{80} - 2(\alpha^2 + \beta^2)] \Theta_4 + \right. \\
& \left. + (F_{54} + F_{61}) \Theta_4' - (F_{68} - F_{75}) \Theta_3' \right\}, \\
\Phi_{30} = & \frac{1}{2r'} \left\{ 2 \frac{\cos^2 \delta_0}{\sin \delta_0} \left[\sum_{i=92}^{94} F_i + F_{95} \Theta_5 + F_{96} \left(\psi_1 + \frac{\psi_2}{r^2} \right) \right] + F_{85} \Theta_2 - \right. \\
& - [F_{80} + 2(\alpha^2 + \beta^2)] \Theta_1 + (p+r) (\psi_5 F_{86} - \psi_4 F_{81}) + \\
& + (F_{54} - F_{61}) \Theta_1' + (F_{68} + F_{75}) \Theta_2' + (p'+r') [\psi_4' (F_{55} - F_{62}) + \\
& + \psi_5' (F_{69} + F_{76})] + \tau \left[2 \frac{\cos^2 \delta_0}{\sin \delta_0} \frac{\psi_3 F_{96}}{r^2} + F_{85} \Theta_4 - \right. \\
& \left. - [F_{80} + 2(\alpha^2 + \beta^2)] \Theta_3 + (F_{54} - F_{61}) \Theta_3' + (F_{68} + F_{75}) \Theta_4' \right] \Big\}, \\
\Phi_{31} = & \frac{1}{2rr'} \left\{ 2r'^2 \left[\sum_{i=97}^{99} F_i + F_{100} \Theta_5' + F_{101} \left(\psi_1' + \frac{\psi_2'}{r'^2} \right) + \frac{\cos^2 \delta_0}{\sin \delta_0} \times \right. \right. \\
& \times [F_{110} \Theta_2' - F_{115} \Theta_1' + (p'+r') (\psi_5' F_{111} - \psi_4' F_{116})] + \\
& \left. + \tau \left[2\psi_3' F_{101} + \frac{\cos^2 \delta_0}{\sin \delta_0} (F_{110} \Theta_4' - F_{115} \Theta_3') \right] \right\}, \\
\Phi_{32} = & \frac{1}{2rr'} \left\{ 2r'^2 \left[\sum_{i=102}^{104} F_i + F_{105} \Theta_5' + F_{106} \left(\psi_1' + \frac{\psi_2'}{r'^2} \right) \right] + \frac{\cos^2 \delta_0}{\sin \delta_0} \times \right. \\
& \times [F_{110} \Theta_1' + F_{115} \Theta_2' + (p'+r') (\psi_4' F_{111} + \psi_5' F_{116})] + \\
& \left. + \tau \left[2\psi_3' F_{106} + \frac{\cos^2 \delta_0}{\sin \delta_0} (F_{110} \Theta_3' + F_{115} \Theta_4') \right] \right\}, \\
\Phi_{33} = & \frac{1}{2r} \left\{ 2 \frac{\cos^2 \delta_0}{\sin \delta_0} \left[\sum_{i=107}^{109} F_i + F_{110} \Theta_5' + F_{111} \left(\psi_1' + \frac{\psi_2'}{r'^2} \right) \right] + F_{105} \Theta_1' + \right. \\
& + [F_{100} - 2(\alpha'^2 + \beta'^2)] \Theta_2' + (p'+r') (\psi_5' F_{101} + \psi_4' F_{106}) + \\
& + (F_{54} + F_{61}) \Theta_2 + (F_{68} + F_{75}) \Theta_1 - (p+r) [\psi_5 (F_{55} - F_{62}) + \\
& + \psi_4 (F_{69} - F_{76})] + \tau \left[2 \frac{\cos^2 \delta_0}{\sin \delta_0} \frac{\psi_3}{r^2} F_{111} + F_{105} \Theta_3' + [F_{100} - \right. \\
& \left. - 2(\alpha'^2 + \beta'^2)] \Theta_4' + (F_{54} + F_{61}) \Theta_4 + (F_{68} + F_{75}) \Theta_3 \right] \Big\}, \\
\Phi_{34} = & \frac{1}{2r} \left\{ 2 \frac{\cos^2 \delta_0}{\sin \delta_0} \left[\sum_{i=112}^{114} F_i + F_{115} \Theta_5' + F_{116} \left(\psi_1' + \frac{\psi_2'}{r'^2} \right) \right] + F_{105} \Theta_2' - \right. \\
& - [F_{100} + 2(\alpha'^2 + \beta'^2)] \Theta_1' + (p'+r') (\psi_5' F_{106} - \psi_4' F_{101}) + \\
& + (F_{54} - F_{61}) \Theta_1 - (F_{68} - F_{75}) \Theta_2 - (p+r) [-\psi_5 (F_{69} + F_{76}) + \\
& + \psi_4 (F_{55} + F_{62})] + \tau \left[2 \frac{\cos^2 \delta_0}{\sin \delta_0} \frac{\psi_3}{r^2} F_{116} + F_{105} \Theta_4' - [F_{100} + \right. \\
& \left. + 2(\alpha'^2 + \beta'^2)] \Theta_3' + (F_{54} - F_{61}) \Theta_3 - (F_{68} - F_{75}) \Theta_4 \right] \Big\},
\end{aligned}$$

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(39)

$$\begin{aligned}
\Phi_{35} &= \frac{1}{2r} [F_{54}\Theta_2 - F_{68}\Theta_1 + (p+r)(-F_{55}\psi_5 + F_{69}\psi_4) + (F_{54}\Theta_4 - F_{68}\Theta_3)\tau], \\
\Phi_{36} &= \frac{1}{2r} [F_{61}\Theta_2 - F_{75}\Theta_1 + (p+r)(F_{62}\psi_5 - F_{76}\psi_4) + (F_{61}\Theta_4 - F_{75}\Theta_3)\tau], \\
\Phi_{37} &= \frac{1}{2r} [F_{54}\Theta_1 + F_{68}\Theta_2 + (p+r)(-F_{55}\psi_4 - F_{69}\psi_5) + (F_{54}\Theta_3 + F_{68}\Theta_4)\tau], \\
\Phi_{38} &= \frac{1}{2r} [F_{61}\Theta_1 + F_{75}\Theta_2 + (p+r)(F_{62}\psi_4 + F_{76}\psi_5) + (F_{61}\Theta_3 + F_{75}\Theta_4)\tau], \\
\Phi_{39} &= \frac{1}{2r'} [F_{54}\Theta'_2 + F_{68}\Theta'_1 + (p'+r')(F_{55}\psi'_5 + F_{69}\psi'_4) + (F_{54}\Theta'_4 + \\
&\quad + F_{68}\Theta'_3)\tau], \\
\Phi_{40} &= \frac{1}{2r'} [F_{61}\Theta'_2 - F_{75}\Theta'_1 + (p'+r')(F_{62}\psi'_5 - F_{76}\psi'_4) + \\
&\quad + (F_{61}\Theta'_4 - F_{75}\Theta'_3)\tau], \\
\Phi_{41} &= \frac{1}{2r'} [F_{54}\Theta'_1 - F_{68}\Theta'_2 + (p'+r')(F_{55}\psi'_4 - F_{69}\psi'_5) + \\
&\quad + (F_{54}\Theta'_3 - F_{68}\Theta'_4)\tau], \\
\Phi_{42} &= \frac{1}{2r'} [F_{61}\Theta'_1 + F_{75}\Theta'_2 + (p'+r')(F_{62}\psi'_4 + F_{76}\psi'_5) + \\
&\quad + (F_{61}\Theta'_3 + F_{75}\Theta'_4)\tau], \\
\Phi_{43} &= \frac{1}{2r} [F_{80}\Theta_2 - F_{85}\Theta_1 + (p+r)(F_{81}\psi_5 - F_{86}\psi_4) + (F_{80}\Theta_4 - F_{85}\Theta_3)\tau], \\
\Phi_{44} &= \frac{1}{2r} [F_{80}\Theta_1 + F_{85}\Theta_2 + (p+r)(F_{81}\psi_4 + F_{86}\psi_5) + (F_{80}\Theta_3 + F_{85}\Theta_4)\tau], \\
\Phi_{45} &= \frac{1}{2r'} [F_{100}\Theta'_2 - F_{105}\Theta'_1 + (p'+r')(F_{101}\psi'_5 - F_{106}\psi'_4) + \\
&\quad + (F_{100}\Theta'_4 - F_{105}\Theta'_3)\tau], \\
\Phi_{46} &= \frac{1}{2r'} [F_{100}\Theta'_1 + F_{105}\Theta'_2 + (p'+r')(F_{101}\psi'_4 + F_{106}\psi'_5) + \\
&\quad + (F_{100}\Theta'_3 + F_{105}\Theta'_4)\tau], \\
\Phi_{47} &= \frac{1}{2rr'} \left\{ \frac{\cos^2 \delta_0}{\sin \delta_0} [F_{90}\Theta_2 + F_{95}\Theta_1 + F_{110}\Theta'_2 + F_{115}\Theta'_1 + (p+r)(F_{91}\psi_5 + \right. \\
&\quad + F_{96}\psi_4) + (p'+r')(F_{111}\psi'_5 + F_{116}\psi'_4)] - 2r^2[(\alpha s + \beta k) d\Omega + \\
&\quad + (\beta n + \alpha p) di + \beta m d\varepsilon + (\alpha^2 + \beta^2) \Theta_5] - 2r'^2[(\alpha' s' + \beta' k') d\Omega' + \\
&\quad + (\beta' n' + \alpha' p') di' + \beta' m' d\varepsilon + (\alpha'^2 + \beta'^2) \Theta'_5] \Big\}, \\
\Phi_{48} &= \frac{1}{2rr'} \frac{\cos^2 \delta_0}{\sin \delta_0} (F_{90}\Theta_4 + F_{95}\Theta_3 + F_{110}\Theta'_4 + F_{115}\Theta'_3).
\end{aligned}$$

(39)

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Eqs. (37) and (38) are completely exact in contrast to the equations which B. V. Numerov [17] used for this purpose. He took the rectangular heliocentric coordinates of the planet and the earth in the following form:

$$\begin{aligned}x &= r[(1-\sigma)\cos l + \sigma\cos(l-2\Omega)], \\y &= r[(1-\sigma)\sin l - \sigma\sin(l-2\Omega)], \\z &= r\sin i\sin(l-\Omega),\end{aligned}\tag{40}$$

where

$$\sigma = \sin^2 \frac{1}{2} i.$$

Differentiating Eq. (40), it is easy to express changes in the rectangular coordinates by changes in the equatorial elements Ω , i , l , and r . Subsequently, the differentials dx , dy , and dz were introduced into Eq. (3). After uncomplicated but cumbersome transformations the conditional equations for the determination of corrections to the elements of the orbit and systematic errors of the catalogue were obtained. The final equations for $\Delta\alpha_0$ and $\Delta\delta_0$ in [17] contain tens of terms and are unsuitable for analysis in view of their complexity. Therefore, B. V. Numerov separates out the terms with small coefficients, neglecting small terms of the second order in the inclinations i , i' , and δ_0 . The approximate differential equations obtained in this way are subsequently used for the theoretical discussions. In general, the latter lead to the conclusion that observations of minor planets can be used successfully for the combined determination of corrections to the elements of the orbits of the planet and the earth and also of systematic errors of the catalogue. In B. V. Numerov's opinion, the observations should continue for six to eight months around opposition.

In the present paper the rectangular heliocentric coordinates are presented by the rigorous relations (4) and (5) through the six Gaussian constants. This subsequently made it possible to perform the necessary transformations and obtain rigorous equations relating the differences $\Delta\alpha_0$ and $\Delta\delta_0$ to the corrections to the elements of the orbits and the systematic errors of the catalogue. The final Eqs. (37) and (38) are written in such a form that all the coefficients for the unknowns are primarily trigonometric functions with simple arguments. Some combinations of corrections to the orbital elements are unknowns in these equations. The radius vectors of the planet and the earth and

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also the time are included to shorten the final form of the equations in Φ_i .

For many minor and major planets the orbits are almost circular; therefore, r and r' can be considered to be constant with an accuracy of a few percent. With this condition the series of functions Φ_i are practically independent of the time, which simplifies the analysis of the equations obtained. If necessary, it is easy to separate the time terms from Φ_i and use them for the clarification of the conditions for separating out the unknowns in Eqs. (37) and (38). Eqs. (37) and (38) differ from the equations of B.V. Numerov [17], which are presented by D. Brouwer and G. Clemence [27] and others, in their basic construction: in the case of a single variable coefficient, for example $\cos(l - l')$, all the corrections to the elements which at least in some way or other are connected with this coefficient, are collected. Each Φ_i is a combination of corrections to the elements of the orbits of the planet and the earth; some of the unknowns in this or the other Φ_i are predominant, i.e., the coefficients in them are many times larger than the coefficients in others. In the papers of some authors, as a rule, the equations are formulated in such a way when a correction for some element or other enters as an unknown and its coefficient in it is a complicated function composed of several terms. In this approach the original relations between $\Delta\alpha_0$ and $\Delta\delta_0$ and the corrections to the elements of the orbits of the earth and the planet are unsuitable for an investigation of the separation of the unknowns.

Analysis of the Equations and Conclusions

One can obtain a more rigorous representation for the determination of the elements of orientation of fundamental catalogues only if Eqs. (37) and (38) are used in the analysis of the equations for the right ascensions and declinations of the major and minor planets. The determination of the arguments of the trigonometric functions amounts primarily to the calculation of heliocentric longitudes l and l' for each instant of observation. The Gaussian constants $\alpha, \beta, \gamma, \alpha', \beta', \gamma', A, B, C, A', B', C'$ for the selected planet and the earth remain constants and can be calculated once and for all for each planet, since, according to (14) - (24), they are functions only of the elements of the orbits. The functions Φ_i in many cases are practically independent of the time.

We now move on to the analysis of Eq. (37). With this aim we discuss a simple equation which is obtained after separating out terms with the unknowns α and Φ_{21} :

$$\frac{2\rho^2}{rr'} \cos^2 \delta_0 \Delta \alpha_0 = \frac{2\rho^2}{rr'} \cos^2 \delta_0 \alpha + \Phi_{21} \quad (41)$$

or

$$\Delta \alpha_0 = \alpha + \frac{rr'}{2\rho^2 \cos^2 \delta_0} \Phi_{21}. \quad (42)$$

We transform the coefficient of Φ_{21} in the last equation

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$$\begin{aligned} \rho^2 &= r^2 + r'^2 - 2rr' \cos(l-l'), \\ \frac{rr'}{2\rho^2 \cos^2 \delta_0} &= \frac{rr'}{2(r^2 + r'^2)} \left[1 - \delta_0^2 + \frac{2rr'}{r^2 + r'^2} \cos(l-l') - \right. \\ &\quad \left. - \frac{2rr'\delta_0^2}{r^2 + r'^2} \cos(l-l') + \frac{4r^2r'^2}{(r^2 + r'^2)^2} \cos^2(l-l') + \dots \right]. \end{aligned} \quad (43)$$

After substitution of (43) into (42)

$$\begin{aligned} \Delta \alpha_0 &= \alpha + \frac{rr'}{2(r^2 + r'^2)} \Phi_{21} - \frac{rr'\delta_0^2}{2(r^2 + r'^2)} \Phi_{21} + \frac{r^2r'^2}{(r^2 + r'^2)^2} \cos(l-l') \Phi_{21} - \\ &\quad - \frac{2r^3r'^3}{(r^2 + r'^2)^3} \cos 2(l-l') \Phi_{21} + \frac{r^2r'^2\delta_0^2}{(r^2 + r'^2)^2} \cos(l-l') \Phi_{21} + \dots \end{aligned} \quad (44)$$

For the majority of the minor and major planets used in the orientation of the fundamental catalogues the eccentricities are small quantities; therefore, the radius vectors r of the planets vary little with the time. The coefficient $rr'/2(r^2 + r'^2)$, initially multiplied by r^2/rr' (coefficients of Φ_{21}), on the whole also depends weakly on the time. Its value varies by less than 10% during observation of the planets along an entire orbit. On the basis of the coefficient of the term $[rr'/2(r^2 + r'^2)] \Phi_{21}$ in Eq. (44), one can with sufficient accuracy treat it as a constant. Consequently, the unknowns α and part of the term Φ_{21} can be combined into a single common unknown:

$$\alpha + \frac{rr'}{2(r^2 + r'^2)} \Phi_{21} = \Phi_0. \quad (45)$$

Then Eq. (42) should be written as follows:

$$\Delta\alpha_0 = \Phi_0 - \frac{rr'\delta_0^2}{2(r^2+r'^2)^2} \Phi_{21} + \frac{r^2r'^2}{(r^2+r'^2)^2} \cos(l-l') \Phi_{21} -$$

$$- \frac{r^2r'^2\delta_0^2}{(r^2+r'^2)^2} \cos(l-l') \Phi_{21} + \frac{2r^3r'^3}{(r^2+r'^2)^3} \cos^2(l-l') \Phi_{21} \dots \quad (46)$$

In view of the smallness of the eccentricities and the comparatively small declinations of the planets at oppositions the coefficients of δ_0^2 are small in comparison with the coefficient of Φ_0 , and the third coefficient is correlated with the coefficient of Φ_1 . The last term, which is not distinguished in the conditional equations, is not readily separated from Φ_0 . The conditions of separation are worse as the planet is observed nearer to opposition. Thus, one can conclude from what has been stated that the correction of the equinox, $\alpha = \Delta\alpha$, is not readily separated from the corrections to the elements of the orbits of the planet and the earth $d\Omega$, di , dM_0 , dM'_0 and $d\epsilon$. It is not difficult to see that dM_0 and dM'_0 enter into Φ_0 with large coefficients, which for all the planets are an order of magnitude larger in absolute value than for $d\Omega$, di and $d\epsilon$. The conclusion is that dM_0 and dM'_0 are not readily separated in the conditional equations based on the right ascension, are not new, and do not require comment. Concerning the corrections to the longitude of the node $d\Omega$, the inclination of the planet's orbit to the ecliptic di , and the inclination of the ecliptic to the equator $d\epsilon$, it has not been mentioned in one of the papers known to us that these corrections with small coefficients are combined with the correction of 27 the equinox, as a result of which the solution of the systems of conditional equations for the determination of $\Delta\alpha$ gives a certain value which is a combination of several unknowns. In such a case various planets should give inconsistent values of the elements of orientation, which have been detected in practice.

It is easy to understand the appearance of terms in Φ_{21} with di and $d\epsilon$. With this aim we return to Eqs. (25) and (26). The terms $r/r' \cos(A - B) X_5$ and $r/r' \sin(A - B) X_7$ are contained on the right side of Eq. (25), and Eq. (26) has the term Y_0 which contains, as is evident from (27) and (28), the unknowns $d\Omega$, di , and $d\epsilon$. Subsequently, these corrections to the orbital elements entered into Φ_{21} and Φ_{47} . If we discuss the case where a major or minor planet moves in the plane of the ecliptic, i.e., $i = 0$, $\Omega = 0$, then according to (14) - (24) and (39) the coefficients of $d\Omega$, di , and $d\epsilon$ in Φ_{21} are equal to zero. Precisely this case was analyzed by B.V. Numerov in [17]. Therefore, in his opinion, the correction of the equinox in the equations

for α is poorly separated only from dM_0 and dM'_0 . B. V. Numerov arrived at such a conclusion because the final equations used in [17] for the theoretical analysis were approximate, since a series of terms were discarded as small terms of the second order. All terms with the multiplier $\sigma = \sin^2 i/2$ in the equations of B.V. Numerov were dropped. Meanwhile, $\sigma \simeq 0.09$ for the planets with large orbital inclinations to the ecliptic (for example, $i = 35^\circ$). The corrections to the orbital elements $d\Omega$, di , dM_0 , $d\varphi$, $d\omega$, $d\mu$ are not strictly quantities of the first order, but even exceed in absolute value the main unknowns. As an example, the values for six minor planets quoted from the papers [20-23] are presented in Table 1. As is evident from the table, the corrections

TABLE 1

Planet	$\Delta\Omega$	Δi	$\Delta\epsilon$
Nemausa	2".53	0".08	0".14
Eros	-0 .63	-0 .06	-0 .43
Juno	0 .46	-0 .20	-0 .34
Pallas	-1 .20	0 .05	0 .13
Ceres	1 .63	0 .83	0 .86
Vesta	-0 .83	0 .05	0 .07

to the elements can differ significantly from each other. This indicates that the terms of the conditional equations multiplied by σ are of various orders of magnitude. Therefore it is impossible to state that all the terms with σ in the equations of B.V. Numerov are of the second order of smallness as in [17]. Using the data of Table 1, it is easy to establish that some terms with σ are sufficiently large and comparable with the quantities being determined, and consequently it is necessary to keep them in the final equations for analysis. This situation subsequently led to the incomplete conclusions drawn by B.V. Numerov.

To simplify calculations based on the Eq. (39) we present in Table 2 necessary data; the number of the planet is indicated in the first column of Table 2.

TABLE 2

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No.	Name of Planet	α	β	γ	k	m	n	s	p
1	Ceres	0.953	0.909	0.416	0.077	-0.409	-0.031	-0.005	-0.173
2	Pallas	0.998	0.981	0.109	-0.013	0.192	-0.192	0.010	-0.007
3	Juno	0.999	0.983	0.300	0.007	-0.184	0.182	0.009	-0.007
4	Vesta	0.993	0.930	0.389	0.013	-0.384	0.196	0.001	-0.116
5	Hebe	0.986	0.978	0.322	0.033	-0.205	0.164	0.033	-0.168
6	Iris	0.996	0.925	0.303	-0.036	-0.379	0.081	-0.002	-0.093
7	Parthenope	0.997	0.935	0.380	0.023	-0.354	0.211	0.003	-0.054
8	Melponeme	0.996	0.968	0.335	0.021	-0.251	0.222	0.013	-0.043
9	Laetitia	0.998	0.971	0.328	0.016	-0.238	0.222	0.012	-0.027
10	Harmonia	0.997	0.920	0.397	0.029	-0.391	0.010	0.000	-0.075
11	Eros	0.988	0.873	0.441	-0.079	-0.479	-0.239	0.016	-0.128
12	Mercury	0.996	0.882	0.431	0.044	-0.469	-0.306	-0.007	-0.065
13	Venus	0.998	0.912	0.404	0.024	-0.410	-0.089	-0.001	-0.036
14	Mars	1.000	0.909	0.406	0.010	-0.417	-0.270	0.009	-0.018
15	Jupiter	1.000	0.919	0.397	0.009	-0.394	0.065	0.000	-0.022
16	Saturn	0.999	0.924	0.392	0.015	-0.382	0.149	0.001	-0.037
17	Uranus	1.000	0.916	0.400	0.005	-0.402	-0.115	0.000	-0.015
18	Neptune	1.000	0.925	0.390	0.009	-0.379	0.251	0.000	-0.018
19	Pluto	0.961	0.956	0.376	0.078	-0.280	0.123	0.028	-0.262
20	Planet	1.000	0.917	0.398	0.000	-0.398	-0.398	0.000	0.000

No.	Name of Planet	w	q	t	A	B	C	S	P
1	Ceres	0.493	-0.155	0.895	170.8	85.1	71.1	1.016	-0.029
2	Pallas	1.795	-0.249	-1.737	264.2	173.5	153.4	0.825	0.069
3	Juno	-0.573	-0.051	0.603	260.2	170.7	166.7	0.976	0.039
4	Vesta	0.042	-0.113	0.871	194.1	106.8	96.8	1.007	0.020
5	Hebe	-0.168	-0.200	0.622	230.0	142.0	125.9	0.995	0.130
6	Iris	0.045	0.688	0.891	350.0	257.7	265.5	1.004	-0.017
7	Parthenope	-0.378	-0.064	0.870	215.1	126.5	121.0	1.002	0.038
8	Melponeme	-0.513	-0.100	0.726	240.4	151.7	143.6	0.992	0.077
9	Laetitia	-0.575	-0.083	0.706	247.3	158.3	151.7	0.988	0.065
10	Harmonia	0.095	-0.069	0.905	184.0	95.8	89.7	1.003	0.005
11	Eros	0.761	0.121	0.948	34.5	299.4	311.6	1.007	0.089
12	Mercury	0.776	-0.072	0.959	136.8	49.5	42.5	1.000	-0.061
13	Venus	0.379	-0.052	0.924	166.0	77.5	72.8	1.002	-0.014
14	Mars	0.649	-0.021	0.932	139.0	49.6	47.7	1.000	-0.016
15	Jupiter	-0.096	-0.021	0.914	189.0	99.6	97.7	1.000	0.064
16	Saturn	-0.257	-0.038	0.901	202.0	113.0	109.6	1.001	0.015
17	Uranus	0.296	-0.012	0.920	163.0	73.3	72.2	1.000	-0.004
18	Neptune	-0.551	-0.022	0.899	221.0	131.6	129.6	1.000	0.016
19	Pluto	0.355	-0.269	0.711	199.8	114.9	91.0	1.036	0.038
20	Planet	0.917	0.000	0.917	90.0	0	0	1.000	0.000

No.	Name of Planet	K	M	N	Q	T	W	μ	ω
1	Ceres	0.989	0.220	0.458	0.963	1.052	-2.028	771"	72°
2	Pallas	0.933	0.072	-0.022	-6.501	5.893	-4.022	769	310
3	Juno	0.933	0.040	0.030	0.814	0.433	-0.752	814	2.6
4	Vesta	0.978	0.139	0.378	0.967	0.793	-2.355	978	1.9
5	Hebe	0.921	0.175	0.131	0.797	1.615	-2.425	939	238
6	Iris	0.987	-0.110	-0.402	0.981	-0.611	-2.333	963	144
7	Parthenope	0.979	0.075	0.305	0.976	0.455	-2.104	924	193
8	Melponeme	0.944	0.094	0.121	0.895	0.781	-1.709	1020	227
9	Laetitia	0.942	0.075	0.091	0.886	0.657	-1.402	770	267
10	Harmonia	0.994	0.088	0.424	0.920	0.475	-2.310	1039	260
11	Eros	1.031	-0.204	-0.486	0.995	-0.798	1.456	2015	178
12	Mercury	1.034	0.115	0.406	1.010	0.480	-1.375	14732	29
13	Venus	1.005	0.069	0.439	0.999	0.352	-2.150	5768	51
14	Mars	1.009	0.029	0.349	1.005	0.144	-1.654	1887	285
15	Jupiter	0.998	0.027	0.423	0.998	0.142	-2.295	299	273
16	Saturn	0.992	0.047	0.381	0.992	0.264	-2.219	120	338
17	Uranus	1.002	0.016	0.420	1.001	0.084	-2.182	42	96
18	Neptune	0.991	0.028	0.306	0.992	0.156	-1.829	22	263
19	Pluto	0.918	0.304	0.278	0.822	1.963	-2.431	14	113
20	Planet	1.000	0.000	0.000	1.000	0.000	0.000	3548	101

We move on now to an analysis of conditional equations in δ .

With this aim we separate out the terms $(2\rho^2/rr')$ α' and Φ_{47} in the Eq. (36);

$$\frac{2\rho^2}{rr'} \Delta\delta_0 = \frac{2\rho^3}{rr'} \alpha' + \text{tg } \delta_0 \Phi_{47} \quad (47)$$

or

$$\Delta\delta_0 = \alpha' + \frac{\text{tg } \delta_0 rr'}{2\rho^2} \Phi_{47}. \quad (48)$$

By analogy with the equation for α after corresponding transformations /29

$$\begin{aligned} \Delta\delta_0 = \alpha' + \frac{rr'}{2(r^2 + r'^2)} \Phi_{47} & \left[\delta_0 + \frac{2rr'\delta_0}{r^2 + r'^2} \cos(l - l') + \right. \\ & + \frac{4r^2r'^2\delta_0}{(r^2 + r'^2)^2} \cos^2(l - l') + \frac{8r^3r'^3\delta_0^2}{(r^2 + r'^2)^3} \cos^3(l - l') + \\ & \left. + \frac{2rr'\delta_0^3}{3(r^2 + r'^2)} \cos(l - l') + \dots \right]. \quad (49) \end{aligned}$$

As in the preceding case, for δ_0

$$\alpha' + \frac{rr'}{2(r^2 + r'^2)} \delta_0 \Phi_{47} = \Phi_0'.$$

Then Eq. (47) is written:

$$\begin{aligned} \Delta\delta_0 = \Phi_0' + \frac{rr'}{2(r^2 + r'^2)} \Phi_{47} & \left[\frac{2rr'\delta_0}{r^2 + r'^2} \cos(l - l') + \right. \\ & + \frac{4r^2r'^2\delta_0}{(r^2 + r'^2)^2} \cos^2(l - l') + \frac{8r^3r'^3\delta_0}{(r^2 + r'^2)^3} \cos^3(l - l') + \\ & \left. + \frac{2rr'\delta_0^3}{3(r^2 + r'^2)} \cos(l - l') \right]. \quad (50) \end{aligned}$$

And so the unknown Φ_0' in Eq. (50) is a function of the corrections of the declinations and the orbital elements of the planet and the earth $d\Omega$, $d\epsilon$, di , dM_0 , $d\varphi$, $d\mu$, $d\varphi'$, $d\mu'$, dM_0' . Consequently, the corrections of the zero point of the equator obtained from the equations where the unknown α' enters with a coefficient close to unity and equal to it (as has been done until now) are also a combination of unknowns.

One can use another criterion for the investigation of the conditionality of systems of linear equations [31].

It is known that in a system of linear equations

$$AX = f \quad (51)$$

a change in the original data, i.e., the matrix A and the vector f, leads to a change in the solution $X = A^{-1}f$. The fact that small relative errors of the system's matrix result in large relative errors in the solution distinguishes a poorly conditioned system. If we take the mean square of its elements as the norm of the matrix A

$$N(A) = \sqrt{\sum_{i=1}^n \sum_{j=1}^n a_{ij}^2} \quad (52)$$

then an inequality occurs for a certain quantity Ω_0

$$\Omega_0 \leq N(A) N(A^{-1}) \Omega_1, \quad (53)$$

where

$$\Omega' = \frac{N(R)}{N(A)} = \frac{\sqrt{\sum_{i=1}^n \sum_{j=1}^n r_{ij}^2}}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n a_{ij}^2}}. \quad (54)$$

The inequality (53) gives a basis for taking the quantity $N(A)N(A^{-1})$ or 30

$$N = \frac{1}{n} N(A) N(A^{-1}). \quad (55)$$

as a measure of the poor conditionality of the system. Such a number N is denoted as the N-conditioning number of the matrix A or the system corresponding to it. Of course, the smaller the number N is, the better conditioned is the system. The best-conditioned matrices, and correspondingly systems, are orthogonal matrices for which the N-conditioning number is equal to unity. Upon the multiplication of all the elements of the matrix by one and the same number, the N-conditioning number does not change, but if one multiplies one column or row of the matrix by a very small or very large number, then the N-conditioning number increases. Certain classes of problems lead to poorly conditioned systems. Since poor conditionality of a system is one of the main reasons for low accuracy of the results of a solution of the systems, then in the case of poor conditionality one resorts to an equivalent transformation of such systems with the purpose of obtaining better conditionality.

We apply this criterion to the system of Eq. (42). There is a common multiplier $1/rr'$ in Φ_{21} which is not equal to zero. After cancellation by $1/rr'$ the matrix of the system of two equations is written as follows:

$$A = \begin{vmatrix} 1 & \frac{1}{2\rho_1^2 \cos^2 \delta_1} \\ 1 & \frac{1}{2\rho_2^2 \cos^2 \delta_2} \end{vmatrix},$$

$$\Delta = \frac{\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2}{2\rho_1^2 \rho_2^2 \cos^2 \delta_1 \cos^2 \delta_2}, \quad (56)$$

and the inverse matrix is

$$A^{-1} = \begin{vmatrix} \frac{\rho_1^2 \cos^2 \delta_1}{\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2} & -\frac{2\rho_1^2 \rho_2^2 \cos^2 \delta_1 \cos^2 \delta_2}{\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2} \\ -\frac{\rho_2 \cos^2 \delta_2}{\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2} & \frac{2\rho_1^2 \rho_2^2 \cos^2 \delta_1 \cos^2 \delta_2}{\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2} \end{vmatrix} \quad (57)$$

It is easy to see that $A \cdot A^{-1} = E$. We find the norms of the matrices A and A^{-1} :

$$N(A) = \frac{\sqrt{8\rho_1^4 \rho_2^4 \cos^4 \delta_1 \cos^4 \delta_2 + \rho_1^4 \cos^4 \delta_1 + \rho_2^4 \cos^4 \delta_2}}{2\rho_1^2 \rho_2^2 \cos^2 \delta_1 \cos^2 \delta_2}, \quad (58)$$

$$N(A^{-1}) = \frac{\sqrt{8\rho_1^4 \rho_2^4 \cos^4 \delta_1 \cos^4 \delta_2 + \rho_1^4 \cos^4 \delta_1 + \rho_2^4 \cos^4 \delta_2}}{\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2}. \quad (59)$$

The value of the conditionality of the system of two equations is

$$N = \frac{8\rho_1^4 \rho_2^4 \cos^4 \delta_1 \cos^4 \delta_2 + \rho_1^4 \cos^4 \delta_1 + \rho_2^4 \cos^4 \delta_2}{8\rho_1^2 \rho_2^2 \cos^2 \delta_1 \cos^2 \delta_2 (\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2)}. \quad (60)$$

If the observations are carried out simultaneously or almost simultaneously, then $N \rightarrow \infty$, since

$$\rho_1^2 \cos^2 \delta_1 - \rho_2^2 \cos^2 \delta_2 \rightarrow 0.$$

If one of the observations is carried out near ($\delta - \delta' = 0$), and the other far from opposition ($\delta - \delta' = 90^\circ$) for a wide range of declinations, then $N > 20$. This denotes that the unknowns α and δ_{21} are uncertainly separated out. One should understand the latter to mean that if the accuracy of the observations, for example, is taken equal to $0''.01$, then α and δ_{21} are determined with a certainty of only $0''.1$. In practice the accuracy of the observations is lower by an order of magnitude; therefore the uncertainty in the determination of the unknown α amounts to approximately a second of arc or, at any rate, several tenths of a second. An increase in the number of observations of a given object is not able to improve the separation of the unknowns to the desired accuracy. /31

Up to this point, we have been dealing with a system of equations with two unknowns. If still other unknowns are included in the equations for α_0 or for δ_0 , the conditions of their separation are worsened. In this case, one should expect still larger errors in the determination of the zero points of the catalogue. In order to lessen these errors, it is desirable to increase the accuracy of the observations in

comparison with the existing accuracy, if only by an order of magnitude. Then and only then one can achieve the results now necessary in the determination of the systematic errors of a catalogue. However, an increase of the accuracy within the indicated limits is a difficult problem and evidently unsolvable within the immediate future.

Thus, it follows from the preceding discussion, that observations of major and minor planets should give inconsistent values of the zero points. The discrepancy of the separate corrections of the equinox and the equator obtained from observations of various planets is completely regular, since a solution of the systems of conditional equations for the right ascensions and declinations by any of the well-known methods is, because of the correlation of coefficients, a certain value which is taken as α or α' . But precisely this quantity, as follows from the equations obtained here, is still a function of the corrections to the orbital elements of the planet and the earth. The inclusion in the conditional equations of a 12th unknown α or α' along with the 11 unknown corrections to the orbital elements cannot be considered justified from the practical point of view, since it is impossible to determine them reliably with the same certainty which is necessary for the creation of fundamental catalogues at the present time. All the advantages of minor planets associated with the convenience and increased accuracy of the observations are lost because of the complicated relations in the conditional equations which it is necessary to solve in order to find the corrections of the equinox, the equator, and the periodic errors of the catalogues.

We move on to the analysis of the conditions of separation of the unknowns b, c in the equations for α_0 , and the unknowns b', c' in the equations for δ_0 . It is not difficult to see that the periodic systematic errors in b, c or b' and c' are theoretically not separable among themselves. Practically, one can consider b and c separable, since, for example, c with a very small coefficient is combined with b in a term with $\cos i$. In rare cases (for certain orbits) b and c, b' and c' are uncertainly determined. The conclusion about the separation of periodic errors of a catalogue is valid only in the case when b and c, b' and c' are of the same order of magnitude. But if these errors are of different orders of magnitude or one is larger than the other by a considerable factor, it is not possible to determine them with certainty.

Thus, if the periodic systematic errors of the catalogues are represented in the form (1), then we find from the solution some combination of the unknowns b and c or b' and c' . If another mathematical representation of the periodic errors is introduced into the analyzed conditional equations, then the necessity of investigating the conditions of their separation arises again. Therefore, the conclusions obtained here about the calculation of the periodic errors of a catalogue are of a special nature.

It is also not difficult to establish that the periodic systematic errors of the catalogue are not readily separated with unknown corrections to the orbital elements. Actually, in Eqs. (37) and (38) there are terms with the coefficients $\sin l$, $\cos l$, $\sin l'$, $\cos l'$, i.e., with almost the same as in the case of b , c , b' , and c' . Applying the methods discussed and the mathematical criteria, it is easy to show that the errors of a catalogue are not readily separated from the corrections to the orbital elements. /32

We note that the terms "poor conditionality of the system" and "poor separation of the unknowns" which are applied here should not be understood in the absolute sense of these words. By "poor separation of the unknowns" is understood here a case where the error of a physical experiment is large for obtaining results of the necessary accuracy. This indicates in our case, that upon the observation of positions of the planets with an accuracy of $0''.2 - 0''.3$ the whole number of seconds in the corrections of the equinox and the equator is reliably determined, but the tenths of a second are uncertainly determined. If the accuracy of the observations is increased by an order of magnitude, then based on the major and minor planets, one can obtain a certain tenth of a second of arc in α and α' . Thus, the values of the zero points are calculated with a certainty of an order of magnitude, or at least a considerable factor worse than the accuracy of the observations themselves. If this satisfies the practical requirements, then the term "poor conditionality of the system" loses meaning. But to determine the coordinates of stars to $0''.01$, for example, positions of the zero points are required with no less reliability.

Naturally, the question arises whether it is impossible to determine from observations a combination of the corrections of the elements and the systematic errors of a catalogue; then knowing the corrections to the

elements, one calculates the error of a catalogue which interests us by process of elimination. A successful solution of the problem in such an arrangement requires obtaining Φ_0 in a pure form, which causes well-known difficulties. Therefore, the necessity arises of selecting a variation for which the function of the corrections of the elements and the errors of a catalogue are determined with certainty. If we put $l - l' = 0^\circ$ in the Eq. (37), i.e., if the planets are observed only at oppositions, which follow one another with respect to longitude, then Φ_{21} and α have practically constant coefficients and their sum is determined with certainty. In this case the Eq. (37) are simplified so that the number of Φ_i becomes less than the number of elements of the orbits of the earth and the planet, and it is not possible to determine the corrections to each element.

One should also note that the presence in the Eqs. (37) and (38) of a large number of unknowns with similar coefficients, places in doubt the reliable determination of the corrections to the orbital elements.

For visualization, the coefficients for the corresponding corrections of the elements in $\Phi_{21} - \Phi_{47}$, and that part of $\Phi_9 - \Phi_{12}$, $\Phi_{29} - \Phi_{30}$, $\Phi_{33} - \Phi_{34}$ which does not clearly depend on the time are presented in Tables 3 - 12. The calculation of the indicated functions Φ_i permits obtaining an idea from the systematic errors of the catalogue about those quantities in the conditional equations which are correlated with the fundamental unknowns a , b , c , a' , b' , and c' . As follows from Table 7, the terms in Φ_{21} with $d\pi$, di , and $d\epsilon$ are not equal to zero for all the major and 11 minor planets. Using the data of Table 1, we find that for Ceres and Pallas Φ_{21} is equal to 1".30 and 0".82, respectively. Frequently, these quantities enter into the equinox correction determined by existing methods, and it is impossible to neglect them, since they exceed α in absolute magnitude. For some fictitious planet with $i = 0^\circ$ and $\Omega = 0^\circ$ (B.V. Numerov's case), which is listed in Table 7 as No. 20, $\Phi_{21} = f(d\Omega, di, d\epsilon) = 0$. This fictitious case obviously permits the determination of the zero points of the catalogue with the necessary accuracy.

Particular attention should be paid to the fact that in Tables 7 and 12 r^2 serves as a multiplier for $d\Omega$, di , and $d\epsilon$. The greater the distance of the planet from the sun, i.e., the smaller the proper motion of the planet, the larger are the terms with $d\Omega$, di , and $d\epsilon$ in Φ_{21} and Φ_{47} . Thus, the more

TABLE 3

$$\Phi_9 = \frac{k_1}{r} dM_0 + k_2 d\varphi + \frac{k_3}{r} d\varphi'$$

No. of Planet	k_1	k_2
1	-0.085	$4.290 \cdot r^{-1} + 0.780$
2	-0.504	$2.773 \cdot r^{-1} + 0.529$
3	-0.514	$-2.629 \cdot r^{-1} - 0.528$
4	+0.184	$1.247 \cdot r^{-1} + 0.266$
5	-0.140	$-4.086 \cdot r^{-1} - 0.878$
6	-0.358	$-3.050 \cdot r^{-1} - 0.675$
7	+0.155	$3.317 \cdot r^{-1} - 0.684$
8	-0.140	$-3.906 \cdot r^{-1} - 0.893$
9	-0.022	$-5.021 \cdot r^{-1} - 0.919$
10	-0.005	$-4.137 \cdot r^{-1} - 0.914$
11	-0.255	$1.383 \cdot r^{-1} + 0.499$
12	-0.072	$-0.170 \cdot r^{-1} - 0.229$
13	-0.004	$0.850 \cdot r^{-1} + 0.588$
14	+0.058	$-2.500 \cdot r^{-1} - 0.828$
15	-0.048	$-9.323 \cdot r^{-1} - 0.898$
16	-0.489	$-0.011 \cdot r^{-1} - 0.001$
17	-0.159	$34.508 \cdot r^{-1} + 0.901$
18	-0.135	$-45.726 \cdot r^{-1} - 0.760$
19	+6.052	$49.720 \cdot r^{-1} + 0.667$
20	-0.016	$0.350 \cdot r^{-1} + 0.175$

Note: 1. In Tables 3-12 the numbers of the planets correspond to the numbers of the planets indicated in Table 2.

2. $k_3 = -0.350$.

TABLE 4

$$\Phi_{10} = \frac{k_1}{r} dM_0 + k_2 d\varphi + \frac{k_3}{r} d\varphi'$$

No. of Planet	k_1	k_2
1	0.178	$1.953 \cdot r^{-1} + 0.355$
2	0.360	$4.411 \cdot r^{-1} + 0.841$
3	-0.386	$4.234 \cdot r^{-1} + 0.851$
4	0.048	$-4.242 \cdot r^{-1} - 0.905$
5	-0.462	$1.588 \cdot r^{-1} + 0.341$
6	-0.389	$2.856 \cdot r^{-1} + 0.632$
7	-0.180	$-2.962 \cdot r^{-1} - 0.611$
8	-0.471	$1.390 \cdot r^{-1} + 0.318$
9	-0.305	$0.493 \cdot r^{-1} + 0.090$
10	-0.094	$0.315 \cdot r^{-1} - 0.077$
11	0.134	$2.203 \cdot r^{-1} + 0.795$
12	-0.014	$0.655 \cdot r^{-1} + 0.883$
13	0.603	$0.988 \cdot r^{-1} + 0.683$
14	-0.118	$-1.184 \cdot r^{-1} - 0.392$
15	-0.228	$2.084 \cdot r^{-1} + 0.201$
16	0.008	$17.596 \cdot r^{-1} + 0.925$
17	0.815	$6.524 \cdot r^{-1} + 0.170$
18	-0.200	$31.582 \cdot r^{-1} + 0.525$
19	6.482	$-54.528 \cdot r^{-1} - 0.732$
20	0.003	$1.800 \cdot r^{-1} + 0.900$

Note: $k_3 = -1.800$.

TABLE 5

$$\Phi_{11} = k_1 dM'_0 + k_2 d\varphi' + k_3 d\varphi(p+r)$$

No. of Planet	k_1	k_2	k_3
1	-0.015	0.528	-1.544
2	-0.016	0.505	-1.096
3	-0.016	0.553	1.138
4	-0.015	0.527	-0.542
5	-0.016	0.513	1.883
6	-0.015	0.529	1.361
7	-0.015	0.530	1.392
8	-0.016	0.535	1.889
9	-0.016	0.542	1.946
10	-0.015	0.526	1.833
11	-0.014	0.474	-0.933
12	-0.014	0.514	0.434
13	-0.015	0.522	-1.170
14	-0.015	0.520	1.641
15	-0.015	0.526	1.800
16	-0.015	0.529	0.000
17	-0.015	0.524	-1.800
18	-0.015	0.530	-1.533
19	-0.015	0.508	-1.404
20	-0.015	0.525	-0.350

TABLE 6

$$\Phi_{12} = k_1 dM'_0 + k_2 d\varphi' + k_3(p+r) d\varphi$$

No. of Planet	k_1	k_2	k_3
1	0.002	2.610	-0.787
2	0.003	2.699	-1.689
3	0.003	2.704	-1.685
4	0.002	2.707	1.771
5	0.002	2.691	-0.575
6	0.004	2.668	-1.314
7	0.002	2.706	1.254
8	0.002	2.705	-0.577
9	0.003	2.707	-0.136
10	0.002	2.708	-0.096
11	0.004	2.627	-1.495
12	0.002	2.711	-1.740
13	0.002	2.708	-1.394
14	0.003	2.705	0.801
15	0.003	2.705	-0.382
16	0.003	2.706	-1.849
17	0.003	2.703	-0.350
18	0.003	2.705	-1.034
19	0.001	2.651	1.264
20	0.003	2.700	-1.800

TABLE 7

/34

$$\Phi_{21} = \frac{1}{rr'} [(k_1 d\Omega + k_2 de + k_3 di + k_4 d\pi)r^2 + k_5 dM_0 + k_6 dM'_0 + k_7 d\pi']$$

No. of Planet	k_1	k_2	k_3	k_5	k_4
1	0.161	0.010	0.381	13.197	1.778
2	0.068	-0.236	0.048	14.629	1.919
3	0.038	-0.089	0.069	13.502	1.923
4	0.110	-0.012	0.377	10.252	1.819
5	0.162	-0.078	0.261	11.107	1.913
6	-0.086	-0.007	-0.392	10.192	1.809
7	0.061	-0.017	0.324	11.137	1.829
8	0.085	-0.062	0.200	9.917	1.893
9	0.069	-0.068	0.155	14.762	1.899
10	0.068	-0.002	0.397	9.421	1.800
11	-0.133	0.043	-0.330	3.560	1.708
12	0.079	0.032	0.291	0.257	1.725
13	0.052	0.007	0.386	0.951	1.784
14	0.022	0.008	0.300	4.203	1.798
15	0.021	-0.002	0.393	49.692	1.798
16	0.037	-0.006	0.369	167.690	1.807
17	0.013	0.002	0.381	673.990	1.792
18	0.022	-0.008	0.301	1672.500	1.809
19	0.254	-0.038	0.377	2791.300	1.870
20	0	0	0	1.834	1.834

Note: $k_6 = k_7 = 1.8337$.

distant the object selected for orientation of the systems of the fundamental catalogues is, the less accurately the zero points are determined. On the basis of what has been said, it is possible to draw the conclusion that the most suitable of all the major and minor planets are Mercury and Venus. Therefore, observations of Mercury, and then Venus, should give the most consistent values for the equinox and equator corrections. It is not difficult to be convinced of this if one compiles a summary of the values of ΔA obtained at various times by different investigators. The advantage of using celestial objects having large proper motions for orienting fundamental catalogues was repeatedly pointed out in the papers [7-12].

We note that the corrections of the equator always have the best agreement, since after dividing the Eq. (38) by $\text{ctn } \delta_0$, we obtain the term $\varphi_{47} \tan \delta_0$. In view of the fact that all of the planets are observed in the near-equatorial zone, this term is small, and consequently the unknowns for the corrections of the elements enter with very small coefficients into the combination $f(\alpha', d\Omega, di, de, dM_0, d\varphi, d\mu, di', dM'_0, d\varphi')$.

TABLE 8

$$\Phi_{28} = k_1 dM_0' + k_2 d\psi' + \left[k_3 d\Omega + k_4 d\varepsilon + k_5 di + k_6 d\omega + k_7 d\mu + \frac{k_8}{r^2} dM_0 \right] \frac{\cos^2 \delta_0}{\sin \delta_0} + k_9 dM_0 + k_{10} d\psi$$

No. of Planet	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
1	-0.016	0.370	0.843	0.554	-1.832	0.820	0	6.261	+0.016 · r^{-1} + 0.168	-0.033 · r^{-1} - 4.265
2	-0.016	0.548	-1.168	2.276	-2.030	0.205	0	1.534	+0.005 · r^{-1} + 1.093	-0.052 · r^{-1} - 2.873
3	-0.016	0.540	0.494	0.190	-0.385	0.599	0	4.118	+0.006 · r^{-1} + 1.164	0.041 · r^{-1} + 2.832
4	-0.016	0.446	0.775	0.394	-1.828	0.772	0	4.289	-0.010 · r^{-1} - 0.373	-0.134 · r^{-1} - 1.274
5	-0.016	0.426	0.591	0.731	-1.445	0.627	0	3.615	-0.014 · r^{-1} + 0.283	0.046 · r^{-1} + 4.381
6	-0.016	0.670	0.784	-0.307	1.834	0.782	0	4.331	+0.041 · r^{-1} + 0.724	0.061 · r^{-1} + 3.086
7	-0.016	0.505	0.749	0.224	-1.542	0.758	0	4.530	-0.014 · r^{-1} - 0.317	0.075 · r^{-1} + 3.384
8	-0.016	0.493	0.618	0.358	-1.024	0.666	0	3.425	-0.006 · r^{-1} + 0.289	0.074 · r^{-1} + 4.133
9	-0.016	0.512	0.594	0.299	-0.810	0.653	0	4.976	-0.004 · r^{-1} + 0.043	0.077 · r^{-1} + 5.321
10	-0.016	0.493	0.794	0.240	-1.843	0.792	0	4.068	-0.002 · r^{-1} + 0.010	0.172 · r^{-1} + 4.164
11	-0.016	0.755	0.902	-0.447	1.474	0.874	0	1.812	+0.019 · r^{-1} + 0.492	-0.168 · r^{-1} - 1.308
12	-0.016	0.466	0.879	0.262	-1.303	0.859	0	0.126	+0.008 · r^{-1} + 0.140	0.024 · r^{-1} + 0.162
13	-0.016	0.507	0.812	0.181	-1.777	0.807	0	0.422	0 · r^{-1} + 0.007	-0.026 · r^{-1} - 0.849
14	-0.016	0.547	0.817	0.075	-1.372	0.812	0	1.877	-0.007 · r^{-1} - 0.115	0.113 · r^{-1} + 2.490
15	-0.016	0.546	0.793	0.071	-1.817	0.794	0	21.463	+0.002 · r^{-1} + 0.098	0.405 · r^{-1} + 9.377
16	-0.016	0.528	0.780	0.131	-1.717	0.783	0	71.161	+0.038 · r^{-1} + 0.988	0.146 · r^{-1} + 0
17	-0.016	0.560	0.801	0.042	-1.754	0.800	0	294.280	+0.018 · r^{-1} + 0.318	-1.498 · r^{-1} - 31.606
18	-0.016	0.546	0.775	0.078	-1.339	0.780	0	705.040	+0.009 · r^{-1} + 0.272	1.957 · r^{-1} + 46.266
19	-0.016	0.285	0.754	0.964	-1.958	0.715	0.011	1090.800	+0.508 · r^{-1} - 12.429	-2.479 · r^{-1} - 52.502
20	-0.016	0.572	0.796	0	0	0.796	0	0.796	+0.001 · r^{-1} + 0.032	-0.015 · r^{-1} - 0.351

TABLE 9

$$\Phi_{30} = k_1 dM_0 + k_2 d\varphi' + \left[k_3 d\Omega + k_4 d\varepsilon + k_5 di + k_6 d\mu + k_7 d\omega + \frac{k_8}{r^2} dM_0 \right] \frac{\cos^2 \delta_0}{\sin \delta_0} + k_9 dM_0 + k_{10} d\varphi$$

No. of Planet	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
1	-0.003	2.445	-0.168	1.914	0.681	-0.001	0.143	1.093	$-0.003 \cdot r^{-1} + 0.329$	$0.209 \cdot r^{-1} + 2.174$
2	-0.003	2.660	-0.944	-2.842	3.088	0	0.073	0.546	$-0.013 \cdot r^{-1} + 0.709$	$0.019 \cdot r^{-1} + 4.429$
3	-0.003	2.656	-0.074	1.219	-1.170	-0.001	0.034	0.238	$+0.011 \cdot r^{-1} - 0.786$	$0.022 \cdot r^{-1} + 4.195$
4	-0.003	2.509	-0.130	1.806	-0.146	-0.001	0.098	0.542	$-0.012 \cdot r^{-1} + 0.114$	$-0.113 \cdot r^{-1} - 4.161$
5	-0.003	2.647	-0.273	1.447	-0.681	0	0.146	0.841	$+0.010 \cdot r^{-1} - 0.927$	$-0.068 \cdot r^{-1} + 1.339$
6	-0.003	2.495	0.101	1.820	-0.086	-0.001	0.075	-0.417	$+0.015 \cdot r^{-1} - 0.750$	$0.170 \cdot r^{-1} + 2.979$
7	-0.003	2.524	-0.076	1.760	-0.866	-0.001	0.053	0.317	$+0.008 \cdot r^{-1} - 0.352$	$-0.137 \cdot r^{-1} - 3.048$
8	-0.003	2.616	-0.132	1.501	-1.147	0	0.075	0.384	$+0.017 \cdot r^{-1} - 0.947$	$-0.026 \cdot r^{-1} + 1.263$
9	-0.003	2.624	-0.112	1.446	-1.230	-0.001	0.061	0.462	$+0.009 \cdot r^{-1} - 0.610$	$-0.040 \cdot r^{-1} + 0.372$
10	-0.003	2.483	-0.079	1.833	0.052	-0.001	0.060	0.306	$+0.008 \cdot r^{-1} - 0.189$	$-0.051 \cdot r^{-1} + 0.217$
11	-0.003	2.354	0.124	1.972	1.339	0	-0.117	-0.242	$-0.040 \cdot r^{-1} + 0.307$	$0.082 \cdot r^{-1} + 2.096$
12	-0.003	2.377	-0.075	1.944	1.454	0	0.068	0.010	$+0.005 \cdot r^{-1} - 0.035$	$0.036 \cdot r^{-1} + 0.650$
13	-0.003	2.461	-0.059	1.861	0.660	0	0.045	0.024	$-0 \cdot r^{-1} + 0.006$	$0.056 \cdot r^{-1} + 1.012$
14	-0.003	2.454	-0.024	1.866	1.267	0	0.018	0.043	$+0.011 \cdot r^{-1} - 0.236$	$-0.070 \cdot r^{-1} - 1.215$
15	-0.003	2.481	-0.024	1.830	-0.233	-0.002	0.018	0.487	$-0.020 \cdot r^{-1} - 0.459$	$0.040 \cdot r^{-1} + 1.991$
16	-0.003	2.494	-0.043	1.809	-0.586	-0.004	0.033	2.984	$+0.008 \cdot r^{-1} + 0$	$0.689 \cdot r^{-1} + 17.636$
17	-0.003	2.473	-0.013	1.841	0.568	-0.013	0.011	4.105	$-0.071 \cdot r^{-1} + 1.634$	$0.378 \cdot r^{-1} + 6.730$
18	-0.003	2.497	-0.025	1.800	-1.137	-0.024	0.019	17.238	$+0.017 \cdot r^{-1} - 0.404$	$0.994 \cdot r^{-1} + 31.187$
19	-0.003	2.575	-0.321	1.808	0.110	-0.034	0.232	354.430	$-0.652 \cdot r^{-1} + 13.808$	$1.933 \cdot r^{-1} - 47.258$
20	-0.003	2.476	0	1.834	1.834	0	0	0	$0 \cdot r^{-1} + 0.006$	$0.078 \cdot r^{-1} + 1.807$

TABLE 10

$$Q_{23} = \frac{k_1}{r} dM_0 + k_2 d\varphi - 0.796(d\Omega' - d\omega') +$$

$$+ dM'_0 \frac{\cos^2 \delta_0}{r \sin \delta_0} + 0.032 \frac{dM'_0}{r} - 0.397 \frac{d\varphi'}{r}$$

No. of Planet	k_1	k_2
1	-0.093	$4.678 \cdot r^{-1} + 0.850$
2	-0.549	$3.024 \cdot r^{-1} + 0.577$
3	-0.593	$-2.867 \cdot r^{-1} - 0.576$
4	0.200	$1.360 \cdot r^{-1} + 0.290$
5	-0.153	$-4.456 \cdot r^{-1} - 0.958$
6	-0.390	$-3.327 \cdot r^{-1} - 0.736$
7	0.169	$-3.618 \cdot r^{-1} - 0.746$
8	-0.153	$-4.259 \cdot r^{-1} - 0.974$
9	-0.024	$-5.476 \cdot r^{-1} - 1.002$
10	-0.005	$-4.511 \cdot r^{-1} - 0.997$
11	-0.278	$1.509 \cdot r^{-1} + 0.514$
12	-0.079	$-0.185 \cdot r^{-1} - 0.250$
13	-0.004	$0.227 \cdot r^{-1} + 0.641$
14	0.063	$-2.726 \cdot r^{-1} - 0.903$
15	-0.053	$-10.166 \cdot r^{-1} - 0.979$
16	-0.533	$-0.012 \cdot r^{-1} - 0.001$
17	-0.173	$37.632 \cdot r^{-1} + 0.983$
18	-0.147	$-49.866 \cdot r^{-1} - 0.829$
19	6.600	$54.220 \cdot r^{-1} + 0.728$
20	-0.017	$0.381 \cdot r^{-1} + 0.191$

TABLE 11

$$Q_{34} = \frac{k_1}{r} dM_0 + k_2 d\varphi - [1.834(d\varepsilon + di')] \frac{1}{r} \times$$

$$\times \frac{\cos^2 \delta_0}{\sin \delta_0} + 0.006 \frac{dM'_0}{r} + 1.729 \frac{d\varphi'}{r}$$

No. of Planet	k_1	k_2
1	-0.163	$-1.791 \cdot r^{-1} - 0.325$
2	-0.330	$-4.015 \cdot r^{-1} - 0.772$
3	0.354	$-3.883 \cdot r^{-1} - 0.780$
4	-0.044	$3.890 \cdot r^{-1} + 0.830$
5	0.423	$-1.456 \cdot r^{-1} - 0.313$
6	0.357	$-2.619 \cdot r^{-1} - 0.580$
7	0.165	$2.716 \cdot r^{-1} + 0.560$
8	0.432	$-1.274 \cdot r^{-1} - 0.291$
9	0.280	$-0.452 \cdot r^{-1} - 0.083$
10	0.087	$-0.319 \cdot r^{-1} - 0.071$
11	-0.123	$-2.020 \cdot r^{-1} - 0.729$
12	0.013	$-0.600 \cdot r^{-1} - 0.810$
13	-0.003	$-0.906 \cdot r^{-1} - 0.626$
14	0.109	$1.086 \cdot r^{-1} + 0.359$
15	0.209	$-1.911 \cdot r^{-1} - 0.184$
16	-0.008	$-16.136 \cdot r^{-1} - 0.848$
17	-0.747	$-5.982 \cdot r^{-1} - 0.156$
18	0.183	$-28.362 \cdot r^{-1} - 0.482$
19	-5.944	$50.002 \cdot r^{-1} + 0.671$
20	-0.003	$-1.651 \cdot r^{-1} - 0.826$

TABLE 12

$$\Phi_{47} = \frac{1}{rr'} \left[(k_1 d\Omega + k_2 di) r^2 + (k_3 r^2 + 0.365) d\varepsilon + k_4 dM_0 \frac{\cos^2 \delta_0}{\sin \delta_0} \right] +$$

$$+ k_5 d\varphi \frac{\cos^2 \delta_0}{\sin \delta_0} + k_6 d\mu + 0.365 di' + 0.001 dM'_0 + 1.172 \frac{d\varphi'}{r} \cdot \frac{\cos^2 \delta_0}{\sin \delta_0}$$

No. of Planet	k_1	k_2	k_3	k_4	k_5	k_6
1	-0.065	0.199	0.372	-0.071	$-1.379 \cdot r^{-1} - 0.252$	0.001
2	-0.027	0.195	-0.188	-0.017	$-0.572 \cdot r^{-1} - 0.109$	0.002
3	-0.016	-0.171	0.181	0.130	$-1.229 \cdot r^{-1} - 0.247$	0.001
4	-0.044	0.017	0.357	-0.034	$1.659 \cdot r^{-1} + 0.356$	0.001
5	-0.065	-0.054	0.200	0.161	$-0.104 \cdot r^{-1} - 0.022$	0.001
6	0.035	0.018	0.351	0.144	$-1.387 \cdot r^{-1} - 0.307$	0.001
7	-0.024	-0.143	0.331	0.067	$1.333 \cdot r^{-1} + 0.275$	0.001
8	-0.033	-0.172	0.243	0.169	$-0.276 \cdot r^{-1} - 0.063$	0.001
9	-0.028	-0.189	0.231	0.103	$0.042 \cdot r^{-1} + 0.008$	0.002
10	-0.027	0.038	0.360	0.041	$0.041 \cdot r^{-1} + 0.009$	0.001
11	0.053	0.335	0.418	-0.094	$-0.966 \cdot r^{-1} - 0.349$	0.001
12	-0.032	0.335	0.414	0.011	$-0.310 \cdot r^{-1} - 0.418$	0
13	-0.021	0.137	0.374	-0.001	$-0.468 \cdot r^{-1} - 0.324$	0
14	-0.009	0.263	0.379	0.052	$+0.565 \cdot r^{-1} + 0.187$	0.001
15	-0.008	-0.038	0.362	0.099	$-0.766 \cdot r^{-1} - 0.074$	0.004
16	-0.015	-0.101	0.353	0.009	$-7.460 \cdot r^{-1} - 0.392$	0.010
17	-0.006	0.118	0.368	-0.354	$-3.138 \cdot r^{-1} - 0.082$	0.029
18	-0.008	-0.214	0.351	0.086	$-12.631 \cdot r^{-1} - 0.210$	0.056
19	-0.101	0.134	0.268	-3.473	$11.753 \cdot r^{-1} + 0.158$	0.088
20	0	0.365	0.365	-0.001	$-0.781 \cdot r^{-1} - 0.391$	0

In conclusion we note the following. To investigate the conditions of separation of the unknowns in the equations, weighted coefficients are often used. This method was applied by G. Clemence in the paper [29] and does not give rise to objections if all the unknowns which characterize the given process, are contained in the original equations. If there are systematic errors in the measurements, which are represented in the equations by terms with identical coefficients, then upon a solution it is possible to determine only their sum. If the equation is written so that only one of the two terms with identical coefficients is included and the other is absent for the reason that nothing is known about an error of the given type, then upon solution of the system of the conditional equations the total size of the two effects is found, and it is attributed to a single unknown. This sum is found with a large weight, but it does not correspond to the physical essence of the phenomenon. To investigate the conditions of separation of the unknowns of the corrections of orbital elements and the systematic errors in the catalogue, the original equations were written by G. Clemence in such a form when it was difficult to establish a correlation between the coefficients of α and α' and part of the corrections to the elements. As was pointed out above, it exists. Consequently, upon solution, it is possible to find weighted coefficients of some more complicated function $f(\alpha, d\Omega, di, \dots)$ or $f(\alpha', d\Omega, di, \dots)$, and not α or α' . The weighted coefficients of combinations of α , α' , and the corrections to the elements are large, but this does not at all imply that the equinox and equator corrections themselves are reliably determined. /38

Which objects of the solar system satisfy the conditions for the best determination of the equinox and the equator corrections?

On the basis of Eqs. (37) and (38), it is easy to conclude that for the determination of the zero points it is necessary to select orbits with a longitude of the node $\Omega = 0^\circ$.

To derive the equinox correction, celestial objects which move strictly in the ecliptic are suitable. In this case, the equinox correction is found more reliably than in other cases. In nature, not a single object except the sun moves in the ecliptic. However, in the list of the known minor planets it is easy to find those which have a very small inclination of the

orbit to the ecliptic and an ascending node which almost coincides with the point of the vernal or autumnal equinox. Ortud ($\Omega = 8^{\circ}.4$, $i = 0^{\circ}.4$) and Lindemania ($\Omega = 2^{\circ}.9$, $i = 1^{\circ}.2$) can serve as examples of such planets. Unfortunately, these minor planets are not bright, so their observation is more difficult.

For selection of the correction of the declinations, the best objects to consider are those which are more in the plane of the celestial equator. Among objects of the solar system, it is possible to select some faint planets with inclinations to the ecliptic which are close to ϵ and with a descending node almost equal to zero: Eukrate ($\Omega = 0^{\circ}.4$, $i = 25^{\circ}.1$), Erminia ($\Omega = 3^{\circ}.2$, $i = 25^{\circ}.1$), Siva ($\Omega = 1^{\circ}.0$, $i = 22^{\circ}.3$), and Prisma ($\Omega = 1^{\circ}.5$, $i = 23^{\circ}.8$).

The mean daily motion should be considered the main factor in the selection of objects for the orientation of systems of catalogues. The more rapidly an object is moving, the more suitable it is for establishing the equinox point and the equator.

In the future, it is necessary to use for the orientation of the zero points of catalogues artificial satellites of the earth. This is particularly suitable for determining the equator corrections $\Delta\delta_0$. For a 24-hour artificial satellite which is placed in an orbit which coincides with the plane of the equator, the equations for the calculation of $\Delta\delta_0$ are very simple. A stationary satellite can be observed with the help, for example, of a zenith tube mounted at the equator or other astronomical instruments situated at various latitudes. During 24 hours of observations, it is easy to cover half an orbit, which is sufficient for a confident separation of the unknowns in the equations of condition. Thus observations for the determination of the zero points of catalogues would have succeeded in being shortened from a duration of several decades (for the minor planets) to one or a few months (for artificial satellites).

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GENERAL ANALYSIS OF THE RESULTS OF LATITUDE OBSERVATIONS
WITH A DANJON ASTROLABE AT POLTAVA DURING 1961.0 - 1964.4

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ABSTRACT: For processing observations with the Danjon astrolabe at Poltava during the period of 1961.0 - 1964.4 moments of star transitions were calculated using FK3 positions, and 20".47 as the constant of annual aberration. The corrections to be applied to latitudes for transition to the new aberration constant, 20".496, are given.

To decrease the systematic errors latitudes were smoothed by the chain method. The closing error proved to be 0".137. The group corrections are given for transition from the FK3 system to the FK4. 124 normal latitude values were smoothed by the analytical method.

The spectrum of errors was investigated by means of analyzing deviations of the normal latitudes from the smoothed ones. The value S_c 0".000110 was taken as a spectrum characteristic. The amplitude and phase of the annual z -term proved to be unstable.

It was found that the latitude decreases from morning until midnight and then increases again.

Observations with a Danjon astrolabe were begun at Poltava in January of 1961. Stars from the FK3 catalogue were used in making up the program of observations. It is known that the most suitable program which provides equality of the weights for the time and latitude corrections is that which is achieved by fulfilling the condition [1]

$$[\cos^2 A] = [\sin^2 A] \cos^2 \varphi.$$

At the time the program of observations was being made up, there were no accurate clocks at the observatory and therefore there was no sense calculating the time corrections from the results of the observations. In connection with this, stars were chosen in making up the program so as to provide the largest possible weight for determining the latitude. /40

The program of observations consists of six two-hour groups. It was

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compiled in such a way as to have the possibility of observing two groups a night in the course of a year, which is particularly important in the fall and winter for providing sure connections between the groups [2].

In all, during the period 1961.0-1964.4, 666 instantaneous latitudes were derived. Each of these was calculated from the observations of a two-hour group, i.e., in all, 11,322 star transits were observed. The main part of these calculations was carried out on the "Promin'" computer of the Institute of Geophysics of the Ukrainian SSR Academy of Sciences.

To eliminate the effect of the behavior of the recording chronograph on the star observation times, it was investigated by a comparison of these readings with the readings of the Observatory's quartz clocks which came into use during the observations from December 1962. After the introduction of corrections for the chronograph's behavior on the star observation times, all the latitudes were recalculated.

The 1961.0 - 1964.4 series was processed with values of the right ascensions and declinations from the FK3 catalogue and a constant of annual aberration of $20''.47$. The corrections to the latitudes for the transition to the new value, $20''.496$, for the constant of aberration were calculated by the method proposed by B. Gino [3].

Correction equations by groups:

Group	S	Correction
I	δ^h	$-0.391 \sin (\odot - 90^\circ) 0''.026$
II	13	$-0.745 \sin (\odot - 211^\circ) 0''.026$
III	15	$-0.805 \sin (\odot - 234^\circ) 0''.026$
IV	18	$-0.829 \sin (\odot - 270^\circ) 0''.026$
V	22	$-0.777 \sin (\odot - 317^\circ) 0''.026$
VI	0	$-0.696 \sin (\odot - 341^\circ) 0''.026$

(Here S is sidereal time and $\odot$ is the longitude of the sun).

In order to decrease the systematic errors in the latitudes, we smoothed the latitudes obtained.

Initially, the mean values were calculated for each group individually for the complete series of observations of the deviations of the position lines of each star from the circle of altitudes. Then the mean values of the deviations of the position lines μ_i were entered with the sign reversed into the

free terms of the equations

$$x \sin A + y \cos A - dz + \delta h = 0$$

or (which is one and the same) corrections of the following type were introduced into the latitudes:

$$\sigma\varphi = - \frac{\sum \mu_i \cos A_i}{\sum \cos^2 A_i}.$$

By this operation, which we call reduction to the center of the group, a decrease is achieved in the systematic error arising because of gaps in the stars, i.e., complete and incomplete series of observations are actually reduced to a single system.

In order to accomplish the reduction of the latitudes to the center of the system of all the groups, we determine the closure error to be 0".137. Considering the fact that all the groups take part in an equal way in generating the closure error, we distributed it uniformly over the remainder of the groups.

The values of the latitude corrections by group after reduction to the center of the system of the series of observations with the astrolabe in 1961.0 - 1964.4 are as follows:

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Group	Correction $\delta\varphi$	Group	Correction $\delta\varphi$
I	+0".09	IV	-0".08
II	-0".02	V	-0".01
III	-0".03	VI	+0".05

As long as the instants of transit of the stars were obtained on the basis of positions from the FK3 catalogue and it was necessary for us to obtain the latitudes also in the system of coordinates in the FK4 catalogue, we decided to proceed in the following way. In order to calculate the corrections to the latitudes for the transition from the FK3 system to the FK4 system for the quantity δh , which is the difference in zenith distances in the sense of "observed minus calculated," one should add the correction:

$$d(\delta h_i) = -15 \sin A \cos \varphi \Delta d_i + \cos g \Delta \delta_i.$$

The constant correction for the entire assembly of stars in question which formed the group is found from the equation:

$$d\varphi' = - \frac{\sum d(\delta h_i) \cos A_i}{\sum \cos^2 A_i}.$$

The values of the latitude corrections by groups of our program are as follows:

Group	Correction $d\varphi'$	Group	Correction $d\varphi'$
I	-0".04	IV	+0".02
II	-0".10	V	+0".03
III	-0".08	VI	+0".08

We introduce these corrections into the latitudes after carrying out the operation of reduction to the group's center. We again determine the closure error - 0".129. Having distributed the closure error uniformly over the remainder of the groups, we find the values of the corrections of the mean group latitudes relative to the center of the system of all latitudes:

Group	Correction $\delta\varphi'$	Group	Correction $\delta\varphi'$
I	+0".12	IV	-0".11
II	+0".07	V	-0".06
III	+0".03	VI	-0".05

Consequently, if we want to transform the system of the FK3 catalogue to the FK4, it is sufficient to introduce into the final latitudes the corrections $\delta\varphi'$ by groups.

After the introduction of the indicated corrections, the normal values of the latitude were calculated. In all, 124 normal values of the latitude were obtained for each 10 days by two methods:

1. Averaging the latitudes in a 10-day interval; to each normal value of the latitude u_i was assigned a weight which is proportional to the number of observed instantaneous latitudes, assuming them to be of equal accuracy. The system of weights which was assigned to the normal points was selected in the following way.

Number of latitudes in a normal point	Weight P_i	Number of latitudes in a normal point	Weight P_i
1 - 3	1	10 - 12	4
4 - 6	2	13 - 15	5
7 - 9	3	16 - 18	6

2. Actually, the instantaneous latitudes are not equally accurate. A measure of the accuracy of individual instantaneous latitudes are their mean square errors σ , which are calculated from internal agreement of the differences "observed zenith distances minus calculated" for each series, i.e., on the basis of the internal agreement of the results in the series. Therefore, the second system of normal points was obtained as the average weight of the

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observed latitudes with a weight equal to the total of the weights of the latitudes which enter into a normal point. The weight of an instantaneous latitude was taken to be inversely proportional to the square of the mean square error of the obtained value of the latitude. (For example, for $\sigma = 0''.09$, the weight $P_i = 1/10\sigma^2 = 12$).

In cases where there was not a single observation in a 10-day interval, the value taken from the preliminary curve of latitude variations obtained graphically by smoothing was assumed to be the normal value of the latitude, and a minimum weight was assigned to it.

In order to obtain the most probable curve of latitude variations, we used an analytical method, first applied by I.I. Glagoleva for reducing the results of latitude observations [4].

The most probable values u' of the observed u for equally spaced values of the argument were obtained from a solution of a system of n linear equations of the form:

$$\begin{aligned} \varepsilon_1 u_1 &= (1 + \varepsilon_1) u'_1 - 3u'_2 + 3u'_3 - u'_4, \\ \varepsilon_2 u_2 &= -3u'_1 + (10 + \varepsilon_2) u'_2 - 12u'_3 + 6u'_4 - u'_5, \\ \varepsilon_3 u_3 &= 3u'_1 - 12u'_2 + (19 + \varepsilon_3) u'_3 - 15u'_4 + 6u'_5 - u'_6, \\ \varepsilon_4 u_4 &= -u'_1 + 6u'_2 - 15u'_3 + (20 + \varepsilon_4) u'_4 - 15u'_5 + 6u'_6 - u'_7, \\ \varepsilon_5 u_5 &= -u'_2 + 6u'_3 - 15u'_4 + (20 + \varepsilon_5) u'_5 - 15u'_6 + 6u'_7 - u'_8, \\ &\dots \dots \dots \\ \varepsilon_n u_n &= -u'_{n-3} + 3u'_{n-2} - 3u'_{n-1} + (1 + \varepsilon_n) u'_n. \end{aligned} \tag{1}$$

The smoothing was carried out by overlapping series with 45 normal values in each. The system of 45 equations of the form (1) was solved by the method of conjugate gradients. This and the subsequent calculations were carried out on the M-20 computer of the Institute of Cybernetics of the Ukrainian SSR Academy of Sciences by coworkers of the Poltava Gravitometric Observatory, S.N. Korb, E.I. Yevtushenko, and E.A. Slavinskaya.

Three smoothed curves of the oscillation of Poltava's latitude are presented in Figure 1.

Curve 1 was obtained from a smoothing of the normal latitude values found by averaging the latitudes in a 10-day interval. The average weight of the normal values was $p_i = 2.2$, the smoothing coefficient was $\varepsilon_0 = 0.1$,

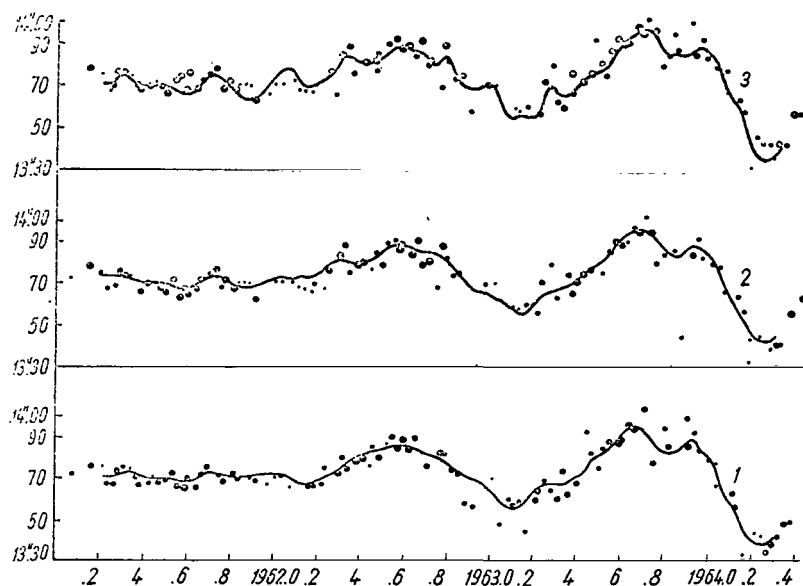


Figure 1. Curves of the oscillations of Poltava's latitude (the diameters of the circles are proportional to the weights of the normal points p_i).

and then $\epsilon_0 p_i = 0.22$.

Curve 2 was obtained from a smoothing of the normal latitudes calculated as the mean weight of the observed latitudes in a 10-day interval. The mean weight of the normal latitude values was $p_i = 2.2$, and $\epsilon_0 p_i = 0.22$.

Two curves of the oscillation of Poltava's latitude were obtained with the same mean smoothing coefficient $\epsilon_0 p_i = 0.22$ or with the same mean smoothing weight in order that they could be compared to one another, since they satisfy with equal probability the normal latitude values. Comparing them to one another, we see that curve 2 differs in particular sections from curve 1 (see Figure 1). Curve 2 is probably more reliable than curve 1 because in assigning the weights of the normal points the internal agreement of the results of observations of individual stars in the series, i.e., the conditions of the observations, was taken into account.

Curve 3 was obtained by smoothing the normal latitudes which were calculated as the mean weights of the observed latitudes with only an average weight of the normal values $p_i = 39.2$, i.e., $\epsilon_0 p_i = 3.92$.

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To estimate the accuracy of the construction of the latitude curve based on the normal points and the determination of the nonpolar latitude oscil-

lations we used the methods of investigation of spectral structure of random sequences, whose method of application to an investigation of a series of latitude observations was worked out by I.I. Glagoleva and Ya.S. Yatskivo [5, 6].

The spectral structure of the smoothed curve of latitude variation is determined from the spectral properties of the transfer function of the linear operator used, whose frequency characteristic is

$$f(\omega) = k_0 + 2 \sum_{p=1}^{\infty} k_p \cos p\omega,$$

where

$f(\omega)$ is a monotonically decreasing function of frequency and k_p depends on the smoothing coefficient ϵ_0 .

The frequency characteristics of the smoothed curves are presented in Figure 2 for the smoothing operators applied by us: 1 - for $\epsilon_0 p_i = 0.22$ and 2 - for $\epsilon_0 p_i = 4.00$. The behavior of the function $|f(\omega)|^2$ (1) shows that $T = 11.68$ 10-day intervals for a value of the quantity $|f(\omega)|^2 = 0.85$.

Because of the dependence

$$S_{sm}(\omega) = |f(\omega)|^2 S_n(\omega),$$

harmonics with periods $T \geq 11.68$ 10-day intervals are preserved in the smoothed curve (1.2) with the same amplitude ($|f(\omega)|^2 \geq 0.85$) at the same time as oscillations with periods of $5.18 < T < 11.68$ enter into it with significantly

smaller spectral densities and the high-frequency oscillations with $T \leq 5.18$ 10-day intervals are not contained at all in the smoothed curve

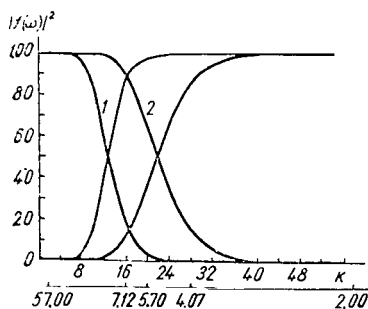


Figure 2. Frequency characteristics of the smoothed curves.

Correspondingly, the frequency characteristic of the smoothed curve with a parameter $\epsilon_0 p_i = 3.92$ shows that in curve 3 there are preserved oscillations with practically the same amount of spectral density as in the original combination of normal values with periods $T \geq 6.83$. Oscillations with periods of $3.08 < T < 6.83$ enter into it with significantly smaller amplitudes, and harmonics with

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$T \leq 3.08$ are not contained in the smoothed curve.

To determine the nature of the errors of observations, we analyzed the differences between the normal latitude values and those taken from the smoothed curve for one and the same epoch.

It is known from the theory of random sequences that the variance of a random sequence is

$$D = \int_0^{\Omega} S(\omega) d\omega,$$

where

$\Omega = 2\pi/2\Delta t$ is the maximum frequency, called the Nyquist frequency, for the sequence given by Δt . Oscillations with a higher frequency, i.e., with a period shorter than $2\Delta t$, cannot be determined from observations carried out during intervals of time Δt , and therefore in the investigation of the spectrum it is necessary to assume that they do not contain significant energy. If this is not so, then we do not take D to be the total variance, but the part of it which arises in the frequency range $(-\Omega, +\Omega)$.

We estimate the effect of errors of observation on the curve of the latitude variation. We calculate an estimate of the spectral density of the energy distribution of the observational errors from the Tukey equation:

$$S_d(\omega_k) = \frac{1}{m} \sum_{\tau=0}^{m-1} \delta_{k\tau} B(\tau) \left(1 + \cos \frac{\pi\tau}{m}\right) \cos \omega_k \tau,$$

where

$$\omega_k = \omega_0 k; \quad \omega_0 = \frac{\pi}{m}; \quad k = 0, 1, 2, \dots, m;$$

$$\delta_{k\tau} = \begin{cases} \frac{1}{2} & \text{for } \tau=0, k=0, \\ 1 & \text{for } \tau=0, k \neq 0, \end{cases}$$

$B(\tau)$ is the correlation function of the deviations of the normal latitude values from the smoothed values corresponding to them and is written as

$$B(\tau) = \frac{1}{n-\tau} \sum_{t=1}^{n-\tau} \Delta\varphi_t \Delta\varphi_{t+\tau},$$

where

$$t = 1, 2, \dots, n$$

for

$$\tau=0, \quad B(0) = \frac{1}{n} \sum_{t=1}^n \Delta\varphi_t^2 = \sigma^2$$

($\Delta\varphi_t$ are the differences of the normal latitude values and the smoothed ones for corresponding epochs). The last expression is the mean variance of the analyzed series.

The properties of the aggregate of deviations of the normal latitude values from the smoothed ones depends not only on the original sequence but also on the type of the operator with whose help the smoothed curve was obtained.

The transfer function $F(\omega)$ of the operator $1 - L$ of the deviations of the normal latitude values from the smoothed ones has the form:

$$F(\omega) = 1 - f(\omega).$$

The spectral density of the deviations of the normal latitude values from the smoothed ones $S_d(\omega_k)$ is associated with the spectral density of the errors of the normal latitude values $S(\omega_k)$ by the relation:

$$S_d(\omega_k) = F^2(\omega) S(\omega_k).$$

Then, knowing $S_d(\omega_k)$, we find the spectral density $S(\omega_k)$ of the errors of the normal latitude values. The lower limit for a level of transmission $F^2(\omega) = 0.85$ for a smoothing coefficient $\epsilon_0 = 0.1$ and a mean weight of the normal values $p_i = 2.2$ is equal to $T \approx 7.12$ 10-day intervals. Then the range of frequencies accessible to investigation is found within the limits:

$$\left. \begin{array}{l} \omega_c = 0.00052\pi \\ \omega_{c'} = 0.00104\pi \end{array} \right\} \quad \text{for the curve with } \epsilon_0 p_i = 3.92,$$

$$\left. \begin{array}{l} \omega_c = 0.00029\pi \\ \omega_{c'} = 0.00104\pi \end{array} \right\} \quad \text{for the curve with } \epsilon_0 p_i = 0.22$$

or for periods, respectively, of 2-7, 12 10-day intervals for the curves 1 and 2 and 2-4.07 10-day intervals for curve 3.

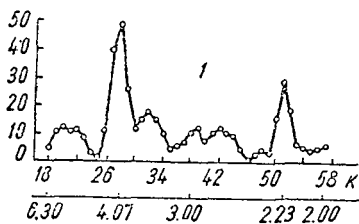
The number of smoothed latitude values for the entire series of observations is equal to 114. The values of $S_d(\omega_k)$ were calculated for a shift of the correlation function $\tau = 0, 1, 3, \dots, 56$.

Since discrete values of the estimate of the function $S(\omega_k)$ are obtained through a frequency interval of $\Delta\omega = 0.00018$, the number of discrete values of the estimates of the function $S(\omega_k)$ in the interval $(\omega_c, \omega_{c'})$ is equal to 42. We take as a characteristic of the spectrum the quantity:

$$S_c = \frac{1}{42} \sum_{k=16}^{57} S(\omega_k) = 0''.000110,$$

which represents the fraction of the variance which enters into a unit frequency interval $\Delta\omega$.

In Figure 3, the sizes of the spectral density of the deviations of the normal latitude values from the smoothed $S_d(\omega_k)$ in units of 0.00001 are plotted along the ordinate axis, and the harmonic numbers k are plotted along the abscissa. The curves 1-3 correspond to the curves 1-3 of Figure 1. The values of the periods in units of 10-days corresponding to the k numbers are presented on the auxiliary abscissa.



T, 10-day intervals

Figure 3. Deviations of power spectra of the normal latitude values from the smoothed ones.

In order to have the possibility of comparing the estimate obtained by us for the accuracy of construction of the curve of latitude oscillations from observations at Poltava with the Danjon astrolabe with the estimates presented in the paper [7], it is necessary to take an identical frequency interval in which the estimates of the function of spectral density of the sequence of accidental errors are determined.

The system of weights which we assign to the normal points is equivalent to the system of weights assumed in the paper [8]; the unit of measurement of the time interval Δt is taken as 0.25 hours. Then, since the successive instantaneous latitudes from the astrolabe observations are obtained from a reduction of a single 2-hour series, such a time interval is equal to 8 unit intervals Δt .

The lower limit of the frequency region in which the spectrum of the accidental errors is determined can be accurately combined with those indicated in the paper [7]. Concerning the upper limit, it is not possible to do this because the averaging intervals differ from each other (in the paper [7]

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weekly averages are taken, but 10-day averages are taken in our calculations).

Therefore, it is possible to make only an approximate comparison of the accuracy of construction of the curve of latitude oscillations based on the normal points from the results of observations at Poltava with the Danjon astrolabe with other series of observations on various instruments, the characteristic of whose accuracy is presented in the paper [7].

	ω_c	$\omega_{c'}$	T
Observations at Poltava	0.00052 π	0.00104 π	20-40.7 days
Data of the paper [7]	0.00052 π	0.00149 π	14-40.6 days

Since the lower limit of the frequencies was shifted, the number of discrete values of the estimates of the function $S(\omega_k)$ in the frequency interval changes. Then

$$S_{c'} = \frac{1}{30} \sum_{k=28}^{57} S(\omega_k) = 0''.000116.$$

A table of values of the accuracy of construction of the curve in the region c for several series of observations is quoted by us from the paper [7]. Comparative characteristics of the spectrum in the frequency region from $\omega_c = 0.00052 \pi$ to $\omega_{c'} = 0.00149 \pi$, with S_c given in units of $0''.000001$, are presented in Table 1.

The differences in latitudes taken from the smoothed curves and the curve constructed from the coordinates of the pole calculated from the data of the MBV* were taken as the original values for the calculation of the z-term. We did not exclude variations of the mean latitude, since the size of such variations did not exceed $0''.01$. The value of the mean latitude for the middle of the interval 1961.0-1964.4 is equal to $13''.75$.

In all, 62 values of the z-term were obtained in 0.05 year. Although peaks for the periods 0.954, 1.127 and 0.954 year for the first, second, and third curves, respectively (Figure 4) were detected by a more detailed spectral analysis, we consider it more suitable to determine the annual wave of the z-term for the three curves (see Figure 1), since for a brief series accidental observational errors could appear. The annual wave of the z-term was found to be equal to:

*International Time Bureau

- 1) $0''.069 \cos (18^\circ t - 23^\circ)$
 ± 3 ± 26 ,
- 2) $0''.058 \cos (18^\circ t - 16^\circ)$
 ± 25 ± 25 ,
- 3) $0''.064 \cos (18^\circ t - 19^\circ)$
 ± 29 ± 26

(t is expressed in units of 0.5 year).

Station	Instrument	Duration of the Observations	S_c
Potsdam	Astrolabe	1958.27-1960.24	88
Paris	Astrolabe	1957.70-1959.67	52
Kazan'	ZTL-180 zenith-telescope	1957.87-1959.84	253
Kitab	ZTL-180 zenith-telescope	1957.78-1959.75	148
Irkutsk	" " "	1958.29-1959.77	95
Poulkovo	ZTF-135 zenith-telescope	1957.62-1959.60	90
Poltava	Zeiss zenith-telescope	1957.62-1959.60	221
"	Bamberg zenith-telescope	1957.62-1959.60	174
Kitab	" " "	1957.78-1959.75	160
Midzusava	" " "	1957.12-1958.89	159
"	Photographic zenith tube	1957.12-1958.89	170
Tokyo	" " "	1958.90-1960.88	58
Richmond	" " "	1958.09-1960.07	245
Washington	" " "	1958.09-1960.07	138
Ottawa	" " "	1958.09-1960.07	177
Greenwich	" " "	1957.83-1959.81	100
Midzusava	Floating zenith-telescope	1957.12-1958.89	102
Dresden	Transit instrument	1957.72-1959.69	161
Poltava	Astrolabe	1961.00-1964.40	116

The calculations showed that the amplitude and phase of the z-term were unstable. The expressions for the z-term were found to be almost identical for the three curves, notwithstanding the fact that curves 1, 2, and 3 (see Figure 4) were obtained with different mean smoothing coefficients ϵ_{0p_i} (0.22 and 3.92, respectively). To investigate the variations of Poltava's latitude

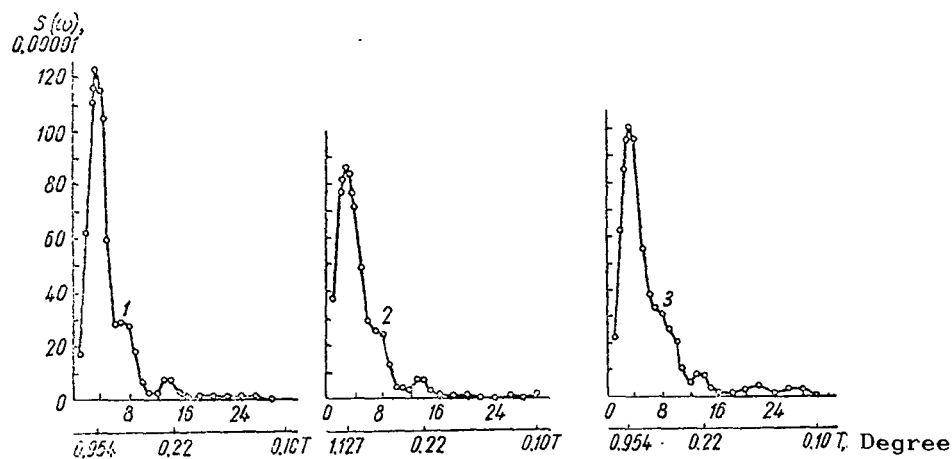


Figure 4. Power spectra of the z-term.

in the course of a night, we divided the entire observational material into groups as a function of the mean time of the observations. The differences in latitudes in the sense "observed latitude minus the latitude taken from the curve constructed from the coordinates of the pole" were found for each group. Then, the mean values of the differences and the average instants of observations were obtained in each group. After correcting the latitudes for the new value of the constant of aberration, the calculations were carried out by the same method. The nature of the curve changed little (Figure 5, curves 2 and 3).

We also investigated the latitude differences observed on one and the same night as a function of the mean time of their observation. The advantage of such a series of investigations consists in the fact that polar and nonpolar variations of long periods are eliminated in the latitude differences, and only latitude variations with a daily period remain. Curve 1 in Figure 5 shows the nature of the latitude variation in the course of a night obtained from two adjacent groups observed on the same night. Since the latitude differences vary almost linearly, it is possible to assume that the nature of the latitude variation during a night can be approximately represented by a curve close to a parabola. In conclusion, the author expresses sincere gratitude to N. I. Panchenko for a series of discussions and advice.

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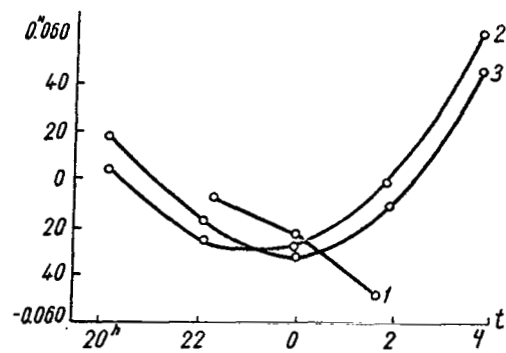


Figure 5. Curves of the latitude variation during a night.

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FREE DIURNAL NUTATION OF THE EARTH BASED ON
OBSERVATIONS AT PULKOVO FROM 1915 TO 1928

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ABSTRACT: The author discusses possibility of studying free diurnal nutation of the earth using customary latitude observations. He gives the results deriving the parameters of this nutation based on the observation at Pulkovo from 1915 to 1928. The period of free diurnal nutation thus obtained is in a good agreement with a theoretical one ($23^{\text{h}}56^{\text{m}}54^{\text{s}}$ sidereal time) calculated by M.S. Molodenskiy for his second model of the earth constitution.

The essential changes of the amplitude and phase of the nutation are revealed. The average value of the amplitude for the series of observations under consideration equals to 0".010.

In the presence of a liquid oblate nucleus the envelope of the earth can have in addition to the Chandler motion of the pole, an additional free nutation with a period of around 24 hours. M.S. Molodenskiy obtained exact values of the period of this nutation for two models of the earth with a liquid core. In particular, the period of free nutation for the second model is equal to $23^{\text{h}}56^{\text{m}}54^{\text{s}}$ of sidereal time [1]. The determination of the parameter of the free diurnal nutation from observations is of great value for refining the model of the interior structure of the earth [2, 3]. Due to the difference of the nutation period from a sidereal day it is possible to include for this purpose observations of the variability of latitude as was done in the papers [4-7]. N.A. Popov obtained the most reliable results. However, the discrepancies between the separate determinations stimulates new attempts of analysis of the material of the astronomical observations for an investigation of the free diurnal nutation of the earth. The method of such an analysis is different for different programs and methods of reduction of the observations of latitude. For observations of bright zenith stars, it is described in the papers of N.A. Popov. Here, we note only that in this case in the sequence of latitudes obtained from the observations of one and the same star on various dates, the free diurnal nutation appears in the form of an oscillation with a period $T_1 = 463.52$ sidereal days (s.d.), which is close to the Chandler period. Therefore, it

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is necessary to exclude from the results of the observations, long-period polar components. It is possible to do this, taking the differences of the latitudes obtained from the observations of two or several stars. Such differences contain, in addition to the nutational, nonpolar variations of the latitude which are caused by diurnal effects. Therefore, for example, in order to separate the annual nonpolar and nutational waves it is necessary to have a series of observations with a duration of not less than

$$\Delta T = \frac{T_2 T_1}{T_1 - T_2} = 1745 \quad \text{s.d.}$$

where

T_1 and T_2 are the periods of the nutational and annual waves.

We assume now that the observations are carried out each night at one and the same instant of mean time. Then, in the sequence of latitudes obtained on various dates the free diurnal nutation appears in the form of an oscillation with a period $T_0 = 204.03$ mean days (m.d.). Actually, the period of nutation for the second model of M.S. Molodenskiy is equal to $23^{\text{h}}52^{\text{m}}58.6^{\text{s}}$ of mean time. From here we find that the velocity of a wave in mean days is $\omega_0 = 361.76443$ to which the period T_0 corresponds. Such an oscillation is located far from the fundamental periodic components of motion of the pole and can be detected in the spectrum of the latitude variations. There is also contained in the latitude differences obtained from evening and morning observations, a nutational wave:

$$\begin{aligned} \varphi_e - \varphi_m &= a_0 \cos(\omega_0 t_e + \beta) - a_0 \cos(\omega_0 t_m + \beta) = \\ &= -2a_0 \sin\left(\frac{t_e - t_m}{2} \omega_0\right) \sin\left(\frac{t_e + t_m}{2} \omega_0 + \beta\right), \end{aligned} \quad (1)$$

where

a_0 is the amplitude of the free nutation;

t_e, t_m are the mean instants of the evening and morning observations expressed in days;

β is the initial phase referred to the mean midnight of the initial epoch of the observations.

Thus, if the observations of latitude are carried out by the usual programs (international type) or at constant instants of mean time, then the errors in the star positions are the main obstacle to the determination of the free diurnal nutation. The provisionally-assumed system of declinations and proper motions of the stars is usually improved on the basis of the same latitude observations. This can lead to a significant distortion of the nutational wave which should be taken into account, proceeding to the determination of its parameters from the usual latitude observations.

General Nature of the Pulkovo Latitude Observations from 1915 to 1928

It follows from (1) that the Pulkovo latitude observations carried out in a comprehensive program can prove to be particularly suitable for the investigation of the free diurnal nutation. Selecting in an appropriate method values of t_e and t_m , it is possible to achieve a significant increase in the amplitude of the nutational wave.

As a result of the combined reduction of four mixed series of Pulkovo observations from 1915 to 1928, A.K. Korol' derived a new system of declinations and proper motions of wide pairs of stars [8]. The latitude corrections were calculated separately for each series from the equation

$$\Delta\delta_i = \frac{1}{n} \sum_{j=1}^n (\varphi_j^o - \varphi_0 - \Delta\varphi_j^c), \quad (2)$$

where

φ_j^o are the observed latitudes of the i -th pair;

φ_0 is the average value of the latitude obtained from all pairs;

$\Delta\varphi_j^c$ are the variations in the latitude of Pulkovo calculated from the coordinates of the pole of A.Ya. Orlov.

After deriving the corrections to the value of a revolution of the screw of the ocular micrometer the corrections of the declinations were found in the second approximation and the averages of four series of observations were taken for each of them. Eq. (2) can be written in the form

$$\Delta\delta_i = \frac{1}{n} \sum_{j=1}^n z_j \cdot \overline{\Delta\delta_i}, \quad (3)$$

where

z_j are the nonpolar latitude variations;

$\overline{\Delta\delta}_i$ is the actual correction of the declination of the i -th pair.

We estimate the contribution of the nonpolar latitude variations to each value, $\Delta\delta_i$. Since one pair in the comprehensive program is observed about every half-year, then replacing the summation by an integration we obtain instead of (3)

$$\Delta\delta_i = \frac{1}{0.5} \int_{-0.25}^{0.25} z(t) dt + \overline{\Delta\delta}_i. \quad (3')$$

The frequency characteristic of such a linear transformation is

$$f_1(\omega) = \frac{1}{\Delta T} \int_{-\Delta T/2}^{\Delta T/2} \cos \omega t dt = \frac{2}{\omega \Delta T} \sin \frac{\omega \Delta T}{2}, \quad (4)$$

where

ΔT is the duration of the observations of a pair.

Then we have for (3')

$$f_1(\omega) = \frac{4}{\omega} \sin \frac{\omega}{4}. \quad (4')$$

It is evident from Figure 1 that the transformation (3') is a low frequency filter - in $\Delta\delta_i$ the contribution of oscillations with periods less than 0.5 year is very small. For an annual nonpolar wave $f_1(2\pi) = 0.636$. The free diurnal nutation appears in z_j in the case of observations of a single star in the form of an oscillation with a period of $T_1 = 463.52$ s.d. For this oscillation

$$\omega = 1.58\pi, \quad f_1(1.58\pi) = 0.767.$$

Thus, the nonpolar variation of latitude and the nutational wave enter into the sequence of corrections of the declinations found from a single annual cycle of the observations with amplitudes multiplied by $f_1(\omega)$. We note that in contrast to the observations of a single star the free diurnal nutation in this sequence appears in the form of an oscillation with the form $T_0 = 204.03$ m.d.

The operation of averaging the declination corrections for all years of the observations can be obtained by a linear transformation (5), which is repeated $v-1$ times (v is the number of annual cycles):

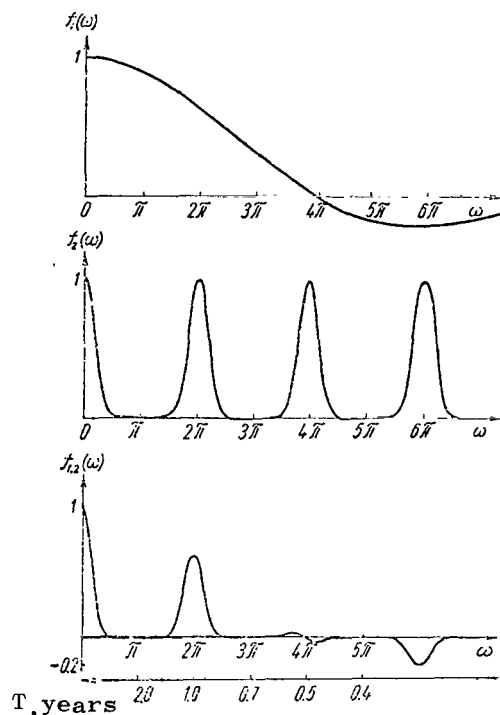


Figure 1. Frequency characteristics of the transformations used in the derivation of corrections of the declinations of latitude pairs.

$$x'(t) = \frac{1}{2} \left[x\left(t - \frac{T}{2}\right) + x\left(t + \frac{T}{2}\right) \right], \quad (5)$$

where

T is the period, which is equal to one year.

The frequency characteristic of such a transformation is

$$f_2(\omega) = \cos^{v-1} \frac{\omega}{2}. \quad (6)$$

For the annual nonpolar and nutational waves /52
we have correspondingly

$$f_2(2\pi) = 1, \quad f_2(3.58\pi) = 0.059.$$

Now, combining (4) and (6), we obtain the frequency characteristic of the transformation applied by A.K. Korol' for finding the declination corrections:

$$f_{1,2}(\omega) = \frac{4}{\omega} \sin \frac{\omega}{4} \cos^{v-1} \frac{\omega}{2}. \quad (7)$$

The form of $f_{1,2}(\omega)$ is shown in Figure 1, from which it is evident that in the sequence of $\Delta\delta_i$ an annual nonpolar wave with an amplitude multiplied by $f_{1,2}(2\pi) = 0.636$ is contained. We find for the multiplier of the amplitude of the free diurnal nutation:

$$f'_{1,2}(\omega_0) = f_1(1.58\pi) f'_2(3.58\pi) = 0.045.$$

Actually, the values of $f_{1,2}(\omega)$ and $f'_{1,2}(\omega_0)$ can differ somewhat from those indicated because of nonuniformity in the distribution and unequal duration of the observations of the pairs. It is evident that in the declination corrections obtained by A.K. Korol' there is contained a contribution of the nonpolar variations, if they exist in the Pulkovo latitude observations. This is confirmed by the disagreement of the declination systems of A.K. Korol' and A.D. Drozd². An annual wave is found in the differences $\Delta\delta_K - \Delta\delta_D$ [8].

2. A.D. Drozd used in his derivation of the declination corrections a preliminary curve of the latitude variation found from the results of the observations.

$$0.021 \cos (\alpha + 126^\circ), \\ \pm 10 \quad \quad \pm 27$$

which, as is evident from Figure 2, is present in the declination corrections derived by A.K. Korol'. Otherwise the power spectra of the declination corrections are very similar to one another. We note that the equation [9] which was /53 used here and in what follows for an estimate of the spectral density is

$$S(k) = \frac{1}{m} \left[\sum_{i=1}^m \gamma_k B(\tau) \left(1 + \cos \frac{\pi}{m} \tau \right) \cos \frac{k\pi}{m} \tau + B(0) \right], \quad (8)$$

where

$B(\tau)$ is the correlation function calculated for the values $0 \leq \tau \leq m$;

$(k = 0, 1, \dots, m)$;

$\gamma_k = 1/2$ for $k = 0$ and $k = m$, $\gamma_k = 1$ for the remaining k .

The diurnal nutational wave is almost not contained in the sequence of $\Delta\delta_i$ values. Its contribution, assuming $a_0 = 0.020$, is less than 0.001 . Consequently, analyzing the instantaneous latitudes, we can find undistorted estimates of the parameters of the free diurnal nutation. The small power of the spectra of the declination corrections in the region of a half-year period evidently confirms the insignificance of the declination errors of such a period and offers the possibility of investigating more reliably the diurnal nutation from the Pulkovo latitude observations.

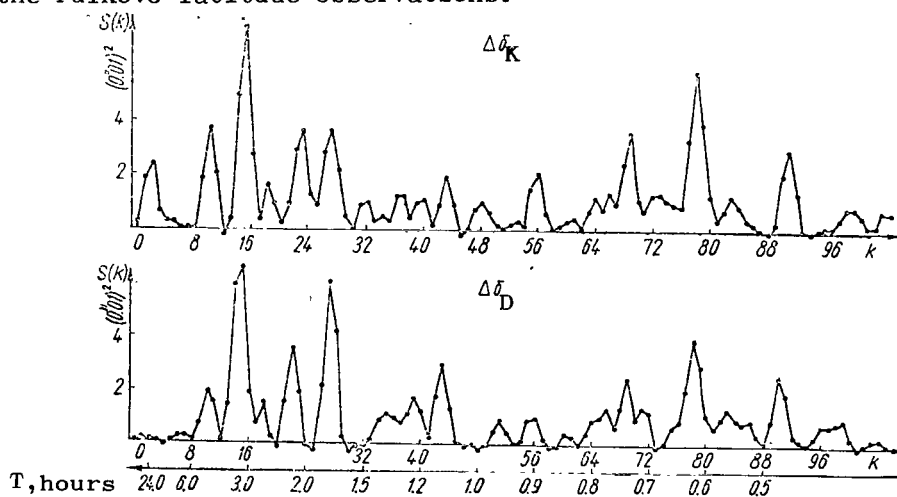


Figure 2. Power spectra of the declination corrections derived by A.K. Korol' ($\Delta\delta_K$) and A.D. Drozd ($\Delta\delta_D$).

Spectral Analysis of the Results of the Latitude Observations (Version I)

A.K. Korol' put at our disposal unpublished instantaneous latitudes corrected for the inaccuracy of the accepted estimated values of a revolution of the screw of the ocular micrometer, declinations, and proper motions. We distributed them according to hours of mean time and selected from the two groups - an evening group and a morning group. In the first version, latitudes obtained from the start of the observations until 22^h30^m were included in the evening group and observations from 1^h30^m until the end of the observations were included in the morning group. Then 10-day means were found separately for each group from the instantaneous latitudes. These means were smoothed by Whittaker's method [10]. The frequency characteristic of the smoothing filter was selected so that harmonics in the range of periods close to the nutational, i.e., 204.03 m.d., were not distorted. Taking into account the fact that the average weight of the normal values p is equal to 5, we selected $\epsilon_0 = 0.1$, $\bar{\epsilon} = \overline{\epsilon_0 p} = 0.5$ (Figure 3). The differences of the smoothed 10-day means of the latitude values which were obtained from the evening and morning groups served as the initial material for an investigation of the free diurnal nutation. In all, there were 476 such differences. We denote them by $\Delta\varphi_I$ and estimate the variation in the amplitude of the nutation wave in $\Delta\varphi_I$ as a result of the formation of:

- a) the means separately for the evening and morning groups,
- b) 10-day normal values,
- c) the differences between the means of the evening and the morning groups.

The duration of the observations of the evening and the morning groups is approximately equal to 3 hours. Then taking $\omega_0 \approx 2\pi$, $\Delta T \approx 0.125$ m.d., we find from (4) $f_a(\omega_0) \approx 0.975$. The transformation (4) is also used for the derivation of the normal values. Here, $\omega = 2\pi/204$, $\Delta T = 10$, $f_B(\omega) = 0.998$. In case (c) the frequency

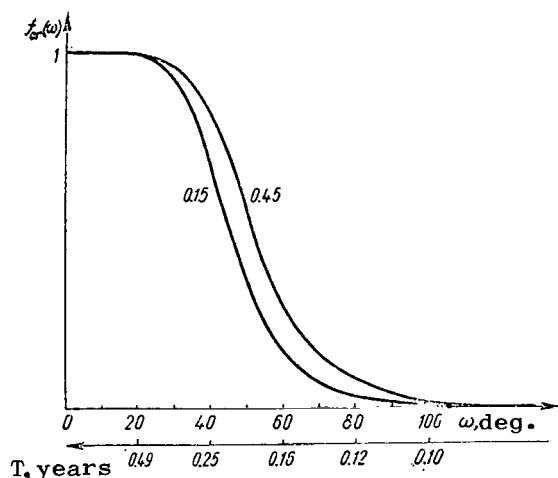


Figure 3. Frequency characteristic of the smoothing filter for the values $\bar{\epsilon} = 0.5$ and $\bar{\epsilon} = 0.45$.

characteristic of the filter of the differences is

$$f_c(\omega) = 2 \sin \omega \frac{\Delta T}{2}.$$

Substituting the values $\omega_0 = 361.76$ and $(\Delta T/2) \approx 0.125$ m.d., we find $f_c(\omega_0) = 1.420$. As a result of the combined action of the transformations a), b), and c), the amplitude of the nutational wave is increased by $M = 1.382$ times.

After the inclusion of the mean value $\Delta\varphi_I \approx -0''.036$, we calculated the power spectrum of the differences with a maximum resolving capability (Figure 4). In the frequency region which interests us, a small peak of spectral density is detected, which it is possible to assign to the free diurnal nutation. We find an approximate estimate of the amplitude of this nutation, having computed the value of the total intensity of the peak

$$E_P = \sum_{k=46}^{48} S(k) = 0.56 \quad \text{in units of } (0''.01)^2 \quad (9)$$

and the continuous spectrum

$$E_C = \frac{1}{4} \sum_{k=44}^{45} S(k) + \frac{1}{4} \sum_{k=49}^{50} S(k) = 0.08 \quad \text{in units of } (0''.01)^2 \quad (9')$$

then

$$\frac{1}{2} a^2 = E_P - E_C = 0.48 \quad \text{in units of } (0''.01)^2$$

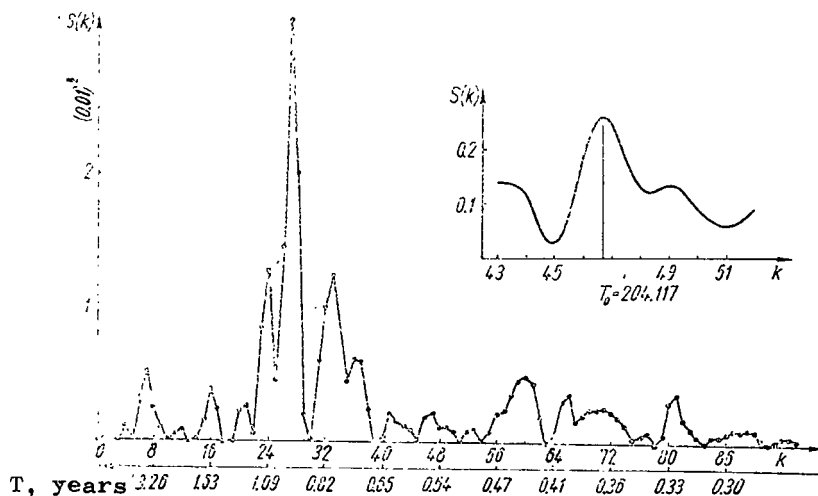


Figure 4. Power spectrum of the latitude differences $\Delta\varphi_I$. In the right upper corner the peak of the nutational wave is shown separately.

Taking into account M, we find the true value of the amplitude $a_0 \approx 0.007$.

We now estimate the size of the period of the free diurnal nutation. With this goal, the values $S(k + \Delta k)$ were found for $\Delta k = 0.1$ (see Figure 4). The maximum spectral density occurs at a value $T_0 = (2m/k) = 204.117$. To this value of T_0 corresponds a period of free diurnal nutation of $23^h 52^m 58.8^s$ of mean time or $23^h 56^m 54.2^s$ of sidereal time, almost coinciding with the value found by M.S. Molodenskiy for his second model of the earth's structure.

Spectral Analysis of the Results of Latitude Observations (Version II)

The mean instants of the evening and morning observations in the first version do not remain constant in the course of a year but vary in accordance with the variation in the duration of night. For this reason there should be present in the power spectrum of $\Delta\varphi_I$ in addition to the fundamental maximum oscillation at the frequency ω supplementary oscillations at frequencies of $\omega \pm \Omega$. Here, Ω is the frequency of a modulating process which describes the variations of the mean instants of the observations of the evening and morning groups. For example, because of the annual variation of the duration of night there appears an annual wave in the differences $\Delta\varphi_I$ if the latitude varies in the course of a night. There should also be detected in the power spectrum of $\Delta\varphi_I$ supplementary maxima of free diurnal nutation situated at the frequencies

$$\omega_0 + \Omega = \frac{2\pi}{130.9}, \quad \omega_0 - \Omega = \frac{2\pi}{462.2}.$$

It is easy to find these peculiarities of the spectrum in Figure 4.

To eliminate the phenomenon described, there were included in the evening group in the second version only those observations which were carried out from $20^h 30^m$ to $22^h 30^m$, and in the morning group - from $1^h 30^m$ to $3^h 30^m$. Maintaining the previous system of weights, we took $\bar{\epsilon} = \epsilon_0 \cdot \bar{p} = 0.3$ for the smoothing of the normal values. The 90% confidence level for $\bar{\epsilon}$, taking into account the fact that in the formation of the smoothed value around 10 unsmoothed normal values participate, is equal to $0.15 < \epsilon < 0.45$. The frequency characteristic of the smoothing carried out does not distort the nutational waves.

In analogy with the first version, we find variations in the amplitude of the free diurnal nutation because of the averaging and the formation of differences

$$f_a(\omega_0) = 0.988, \quad f_b(\omega) = 0.998, \quad f_c(\omega_0) = 1.222; \quad M = 1.207.$$

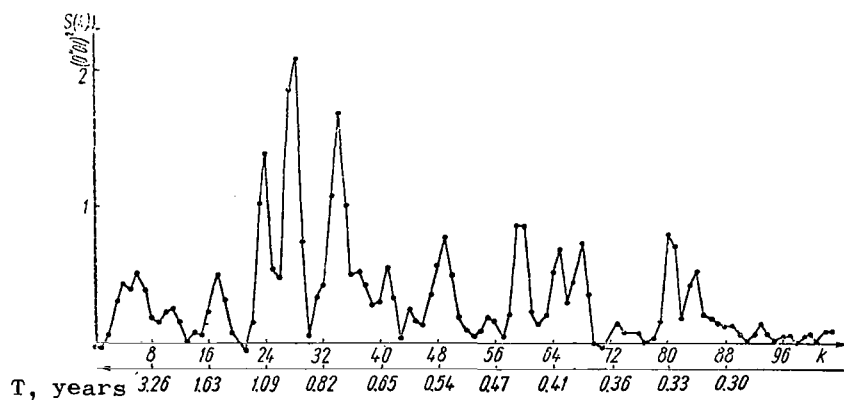


Figure 5. Power spectrum of the latitude differences $\Delta\varphi_{II}$.

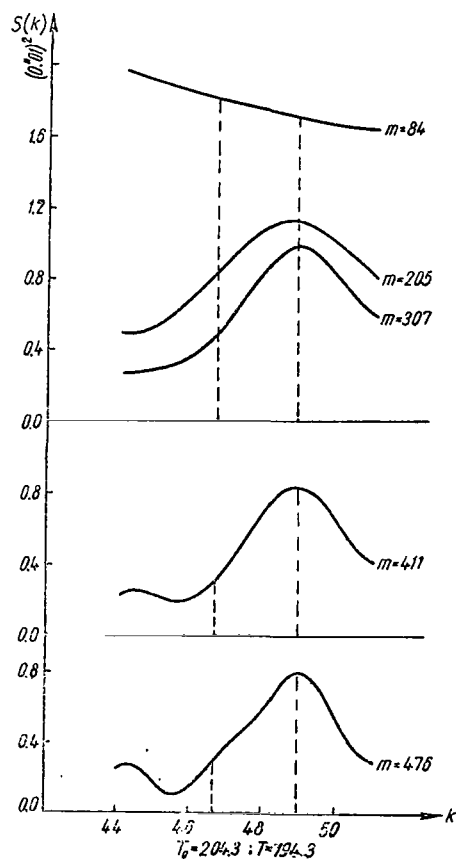


Figure 6. Part of the power spectrum $\Delta\varphi_{II}$ in the frequency region close to the frequency of the nutation for various resolving powers.

After the inclusion of a mean $\Delta\varphi_{II} = -0.035$ we calculated the power spectrum of the differences $\Delta\varphi_{II}$ which is shown in Figure 5. Comparing this with the spectrum of the differences $\Delta\varphi_I$, it is easy to notice a decrease in spectral density at frequencies of $2\pi/365.2$, $2\pi/130.9$ and $2\pi/462.2$. However, in this version the peak of the nutational wave is not separated with a maximum in spectral density which is significant in size, which corresponds to a period of 194.3 m.d. and is barely noticeable in Figure 4. In this region of frequencies, evidently, there are several oscillations, since otherwise the harmonics with periods $T_0 = 204.0$ and $T = 194.3$ m.d. in the spectrum with maximum resolving power would be separated. For a more detailed investigation of these oscillations, the power spectra $\Delta\varphi_{II}$ for various resolving powers were calculated i.e., with a different number of m_p of the values of the correlation function used (Figure 6).

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Let the values of $S_p(\omega)$ of the oscillations which interest us be obtained for a certain value of m_p . Then, according to [11], one can write in the first approximation

$$S_p(\omega) m_p = m_q E_c + \frac{1}{4} m_p a^2 + \zeta(\omega),$$

where

$$p = 1, 2, \dots, q,$$

E_c is the level of the continuous spectrum and

$\zeta(\omega)$ are the accidental deviations.

From Eq. (10) it is possible to determine separately the amplitude of a harmonic and the level of the continuous spectrum of the errors.

For the oscillations with periods $T_0 = 204.0$ and $T = 194.3$ m.d. which interest us, using the data presented in Figure 6 and Eq. (10), we find $a_0^2 = 0$, $E_c = 0.34$, $a^2 = 2.08$, $E_c = 0.27$ in units of $(0.01)^2$. The diurnal nutational wave is not detected by such a method. Consequently, its energy does not exceed the level found for the continuous spectrum, whose significant height is evidently caused by instability in the amplitudes and phases of the oscillations in this frequency region.

Naturally, the question arises about the reality of the oscillations with a period $T = 194.3$ m.d. (the corresponding diurnal period is $23^h 56^m 33^s$ in

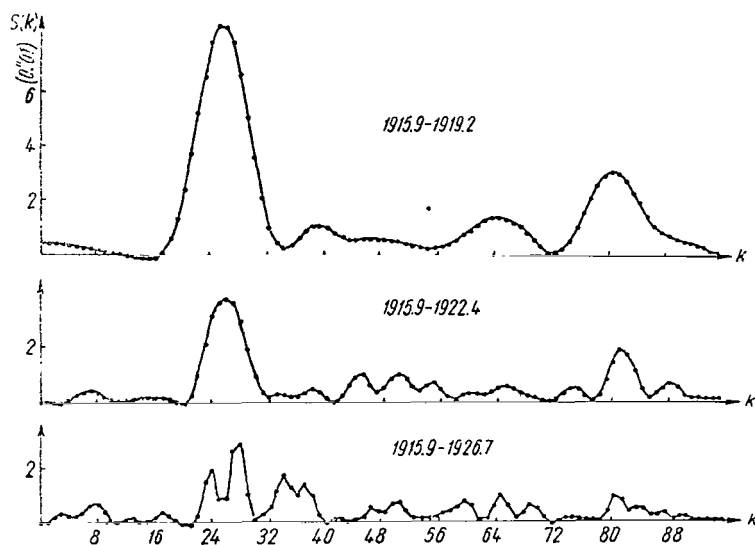


Figure 7. Running power spectrum of the latitude differences $\Delta\varphi_{II}$.

sidereal time). The investigation of the power spectra of the differences $\Delta\varphi_I$ and $\Delta\varphi_{II}$ confirms the dependence of the amplitude of this oscillation on the method of processing the observational results. We discuss now the running spectrum which characterizes the variations in the spectrum of the differences $\Delta\varphi_{II}$ with an increase in the duration of the series of observations. It is evident from Figure 7 that in processing observational material for 1915.9 to 1919.2 the level of the spectral density is almost constant because of the insufficient resolving power of the spectrum. Then two maxima are formed at the frequencies of the fundamental oscillations. The size of the second maximum increases after the addition of observations for 1926.7 to 1928.9, which also confirms the transiency of the oscillation with a period of 194.3 m.d. /58

Thus, the spectral analysis of the differences $\Delta\varphi_I$ and $\Delta\varphi_{II}$ indicates the existence in the frequency range of interest to us of at least two oscillations which have fluctuations in amplitude and phase. The level of the spectral density of these fluctuations is comparable with the peak of the nutational wave. On the basis of the analysis of the observations of a single latitude station it is difficult to judge the reality of the spectral variations detected in the frequency region close to the frequency of the diurnal nutation. They may be an effect of errors in the latitude observations.

Determination of the Parameters of the Free Diurnal Nutation

We obtain the most accurate estimate of the amplitude and phase of the free diurnal nutation from unsmoothed latitude differences derived from the evening and morning observations. With the aim of decreasing the volume of calculation we divided all the evening and morning observations into two 2-hour groups whose mean observational times are

$$t_{1e} = 21^h.5, t_{2e} = 19^h.5, t_{1m} = 2^h.5, t_{2m} = 4^h.5.$$

The differences of the mean values of the corresponding groups served as the initial material for the subsequent analysis. In all, there were 511 differences of the first type, $\Delta\varphi_I$, and 186 of the second type, $\Delta\varphi_{II}$. We now present them in the form

$$\Delta\varphi = z + x \sin \omega_0 t + y \cos \omega_0 t + v(t), \quad (11)$$

where

$z, x = a \cos \beta, y = a \sin \beta$ are unknown quantities and

$v(t)$ is a random function.

With this aim, taking for the initial epoch O^h of local mean time December 1, 1915, the phases $\omega_0 t$ were calculated for each value of $\Delta\varphi$. Then all the differences were grouped according to phase in 30-degree zones and the averages were found for each zone, to which weights proportional to the number of observations were assigned. From a least squares solution of the equations of condition (11) which were set up for these means, results were obtained which are presented in Table 1. The data of Table 1 confirms the statement made above about the instability of the nutational oscillation and the significance of the accidental component.

TABLE

No. of series	Years	Type of differences	(in units of 0.01)					
			z	x	y	σ_z	σ_x	σ_y
1	1915-1918	$\Delta\varphi_1$	-5.81	+3.284	+1.720	+0.62	+0.82	+0.96
2	1919-1921	$\Delta\varphi_1$	-4.75	-1.049	-2.940	+1.65	± 2.31	+2.64
3	1922-1924	$\Delta\varphi_1$	-6.19	+2.067	-1.583	+1.60	+2.27	+2.25
4	1925-1928	$\Delta\varphi_1$	-3.00	+0.978	+1.138	+0.57	+0.79	+0.85
5	1915-1928	$\Delta\varphi_1$	-4.56	+1.221	-0.226	+0.30	+0.41	+0.45
6	1915-1928	$\Delta\varphi_2$	-4.34	+1.994	+0.222	+1.02	+1.35	+1.41

Actually, we find the 90% confidence intervals of x and y for the first series of observations to be $1.78 < x < 4.78$, $-0.04 < y < 3.48$. The values of x and y obtained from the remaining three series do not fall into this interval. Moreover, the width of the 90% confidence interval of x and y found from the observations of the second and third series are two times greater than the width of the same interval for the unknown determined from the observations of the other two series.

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Neglecting the values of the constant term, we present the data of Table 1 in the following form:

- | | |
|---|--|
| 1) $0''.037 \sin (\omega_0 t + 28^\circ)$
$\pm 9 \quad \pm 14$ | 4) $0''.015 \sin (\omega_0 t + 49^\circ)$
$\pm 8 \quad \pm 31$ |
| 2) $0''.031 \sin (\omega_0 t + 250^\circ)$
$\pm 26 \quad \pm 48$ | 5) $0''.012 \sin (\omega_0 t + 350^\circ)$
$\pm 4 \quad \pm 19$ |
| 3) $0''.026 \sin (\omega_0 t + 323^\circ)$
$\pm 23 \quad \pm 51$ | 6) $0''.013 \sin (\omega_0 t + 7^\circ)$
$\pm 9 \quad \pm 27$ |

In order to reduce the latter expression to the system of differences $\Delta\varphi_I$, a multiplier of amplitude 0.661 was taken into account, and the errors σ_a and σ were corrected in a corresponding manner. The average weight of the independent determinations of 5) and 6) is equal to

$$\begin{array}{c} 0''.012 \sin (\omega_0 t + 353^\circ) \\ \pm 4 \quad \pm 19 \end{array} \quad (12)$$

If we take into account the fact that the nutational wave is determined with certainty only from observations in the time period 1915-1918 and 1925-1928 and take the average of 1) and 4), then we obtain

$$\begin{array}{c} 0''.025 \sin (\omega_0 t + 34^\circ) \\ \pm 6 \quad \pm 13 \end{array} \quad (13)$$

In order to reduce these results to the initial wave of the free diurnal nutation which is present in the Pulkovo observations, one should take into account the formation of

- a) the 2-hour mean values for the evening and morning groups,
- b) the means for 30-degree zones, and
- c) the differences of the type $\Delta\varphi_I$.

Using the method described, we find a final amplitude multiplier of $M = 1.193$. Then, in agreement with (1) we have for (12) and (13):

$$\begin{array}{cc} 0''.010 \cos (\omega_0 t + 353^\circ) & \\ \pm 3 & \pm 17 \\ 0''.021 \cos (\omega_0 t + 34^\circ) & \\ \pm 5 & \pm 14 \end{array}$$

Conclusions

From the observations of a single latitude station, it is difficult to judge whether the detected variation in the amplitude and phase of the nutational wave is real. For this reason, we consider the expression obtained from all the observational material as final:

$$\begin{array}{cc} 0''.010 \cos (\omega_0 t + 353^\circ) & \\ \pm 3 & \pm 17 \end{array}$$

We now compare the results obtained by us with the data of N.A. Popov, which were derived from an analysis of observations of bright zenith stars at Poltava. The values of the period of the free diurnal nutation in both cases agree well with the theoretical value found by M.S. Molodenskiy for his second model of the structure of the earth. N.A. Popov also determined several values of the amplitude of the nutation as a function of the interval of the observations. However, each of them is almost twice as large as the mean result presented by us. For example, for the observations of bright zenith stars for 1939-1963 there is given in [5]

$$\begin{array}{cc} 0''.020 \cos (\theta t + 106^\circ) & \\ \pm 4 & \pm 10 \end{array}$$

The initial phase of 106° is referred to 0^h of local sidereal time at Poltava on June 1, 1939. In order to reduce our initial phase to the same epoch, it is necessary to take into account the variations of phase

a) for the time from December 1, 1915, until June 1, 1939

$$\Delta\beta_1 = +23^\circ,$$

b) for the time from mean midnight on June 1 until 0^h sidereal time on the same date, $\Delta\beta_2 = +113^\circ$.

As long as the pole moves clockwise on account of the diurnal nutation, then the differences in longitude of Poltava and Pulkovo can be taken into account. Then the initial phase of the nutational wave found by us, referred to 0^h local time on June 1, 1939, is equal to 129° , which agrees within the limits of accuracy with N.A. Popov's result.

Thus, the results of the analysis of the Pulkovo latitude observations for 1915-1928 confirm the presence of a free diurnal nutation of the earth. The question of the stability of this nutation is still unresolved.

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ON THE PROBLEM OF THE SMOOTHING OF CLOCK CORRECTIONS

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ABSTRACT: The expression is found of the linear transformation operator, the action of which on the observed sequence of clock corrections is equivalent to that of parabolic smoothing adopted by the Time Service of the USSR. The transfer function of the above transformation is derived.

The fluctuations with periods from 35 to 270 days are shown to contribute to both deviations of the observed corrections from the smoothed curve and to the smoothed curve itself.

In the derivation of the system of standard time based on a combination of /61 observed clock corrections, one is obliged to apply this or the other method of determining and smoothing the initial data. In the USSR Time Service, this is done in the following way. First, the means of the corrections of reference time are formed for each two weeks. Then, with the help of parabolic compensation, the smoothed corrections of reference time are found. The deviations of the smoothed values from the observed ones are published in the "Standard Time" bulletins [1].

Any smoothing amounts to a low-frequency filter which preserves in the smoothed sequence only the long-period oscillations. The deviations of the values from the smoothed values were often analyzed with the purpose of discovering in them definite high-frequency components. At the same time there remained unexplained the problem of just which harmonics are contained in these deviations in undistorted form. To estimate this, as follows from the theory of linear transformations, it is necessary to know the frequency characteristics or transfer function of the smoothing operator applied.

Let $\{u_t\}$ be the observed sequence of clock corrections, and $\{u_t^*\}$, the sequence of smoothed values of the clock corrections.

The sequence $\{u_t\}$ is transformed into $\{u_t^*\}$ with the help of the linear smoothing operator L , i.e., one can write:

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$$\{u_i^*\} = L\{u_i\}.$$

It is known that the spectral densities of the original sequence $S(\omega)$ and the smoothed sequence $S^*(\omega)$ are related to one another by the expression

$$S^*(\omega) = |f(\omega)|^2 S(\omega), \quad (1)$$

where

$f(\omega)$ is the frequency characteristic of the operator L , which is sometimes called the transfer function [2].

Consequently, our problem consists of finding the transfer function of the parabolic smoothing operator.

The parabolic method of smoothing the clock corrections is expounded in [3]. The basic equations of this method are as follows:

$$\begin{aligned} u_i^* &= u_{i-1}^* + b_i(t_i - t_{i-1}) + c_i(t_i - t_{i-1})^2, \\ b_i &= b_{i-1} + 2c_{i-1}(t_i - t_{i-1}), \\ c_{i-1} &= \frac{u_i + u_{i+1} - 2u_{i-1}^* - b_{i-1}[(t_i - t_{i-1}) + (t_{i+1} - t_{i-1})]}{(t_i - t_{i-1})^2 + (t_{i+1} - t_{i-1})^2}, \end{aligned}$$

where

$\dots t_{i-1}, t_i, t_{i+1} \dots$ are the epochs of the averaged clock corrections;
 $\dots u_{i-1}, u_i, u_{i+1} \dots$ are the unsmoothed corrections;
 $\dots u_{i-1}^*, u_i^*, u_{i+1}^* \dots$ are the smoothed corrections;
 $\dots b_{i-1}, b_i, b_{i+1} \dots$ are the first time derivatives of the smoothed function u^* ; and
 $\dots c_{i-1}, c_i, c_{i+1} \dots$ are the doubled second time derivatives of the function u^* .

However, if the combination of formulas presented is used for the smoothing, it is not possible to find the transfer function directly. It is necessary to obtain an expression for the operator of the linear transformation L , whose action on the original sequence $\{u_t\}$ is equivalent to the smoothing described by the Eqs. (2) - (4). /62

Let us limit ourselves to consideration of the simple case when the interval $t_{i+1} - t_i$ does not depend on i , and we take this interval as the unit of measurement of time. Then the Eqs (2) - (4) are written in this form:

$$u_i^* = u_{i-1}^* + b_{i-1} + c_{i-1}, \quad (5)$$

$$b_i = b_{i-1} + 2c_{i-1}, \quad (6)$$

$$c_{i-1} = \frac{1}{5}(u_i + u_{i+1}) - \frac{2}{5}u_{i-1}^* - \frac{3}{5}b_{i-1}. \quad (7)$$

Substituting the value of c_{i-1} from (7) into (5) and (6) and carrying out the simple transformations, we find:

$$\left. \begin{aligned} u_i^* &= \frac{1}{5}(u_{i+1} + u_i) + \frac{3}{5}u_{i-1}^* + \frac{2}{5}b_{i-1}, \\ b_i &= \frac{2}{5}(u_{i+1} + u_i) - \frac{4}{5}u_{i-1}^* - \frac{1}{5}b_{i-1}. \end{aligned} \right\} \quad (8)$$

If we replace here the subscript i by $i-1$, we obtain:

$$u_{i-1}^* = \frac{1}{5}(u_i + u_{i-1}) + \frac{3}{5}u_{i-2}^* + \frac{2}{5}b_{i-2},$$

$$b_{i-1} = \frac{2}{5}(u_i + u_{i-1}) - \frac{4}{5}u_{i-2}^* - \frac{1}{5}b_{i-2}$$

and after substitution of these expressions into (8) we have:

$$u_i^* = V_{i-1} + \frac{1}{5^2}u_{i-2}^* + \frac{4}{5^2}b_{i-2}, \quad (9)$$

where

V_{i-1} is a quantity which depends only on the observed time corrections and has the form:

$$V_{i-1} = \frac{1}{5}(u_{i+1} + u_i) + \frac{7}{5^2}(u_i + u_{i-1}).$$

Furthermore, we substitute in (9) in place of the correction u_{i-2}^* its expression in terms of u_{1-3}^* and so forth, until we arrive at the expression:

$$u_i^* = V_{i-p} + \frac{m_p}{5^{p+1}}u_{i-(p+1)}^* - \frac{n_p}{5^{p+1}}b_{i-(p+1)}, \quad (10)$$

where

m_p and n_p are constant coefficients different for various p .

Our problem consists of expressing the smoothed corrections u_i^* by a linear combination of unsmoothed corrections u_i . Therefore, it is necessary to discuss the value of p for which one can neglect the second and third terms in the expression (10). Taking into account the accuracy of the determination of the clock corrections, we have found $p = 12$. Then

$$u_i^* \simeq V_{i-12},$$

where

$$\begin{aligned}
 u_{i-12} = & \frac{1}{5}(u_{i+1}+u_i) + \frac{7}{5^2}(u_i+u_{i-1}) + \frac{9}{5^3}(u_{i-1}+u_{i-2}) - \frac{17}{5^4}(u_{i-2}+u_{i-3}) - \\
 & - \frac{79}{5^5}(u_{i-3}+u_{i-4}) - \frac{73}{5^6}(u_{i-4}+u_{i-5}) + \frac{249}{5^7}(u_{i-5}+u_{i-6}) + \\
 & + \frac{863}{5^8}(u_{i-6}+u_{i-7}) + \frac{481}{5^9}(u_{i-7}+u_{i-8}) - \frac{3353}{5^{10}}(u_{i-8}+u_{i-9}) - \\
 & - \frac{9111}{5^{11}}(u_{i-9}+u_{i-10}) - \frac{1457}{5^{12}}(u_{i-10}+u_{i-11}) + \frac{42641}{5^{13}}(u_{i-11}+u_{i-12}).
 \end{aligned} \tag{11}$$

After simple rearrangement of (11) we obtain:

$$\begin{aligned}
 u_i^* = & +0.200000 u_{i+1} + 0.480000 u_i + 0.352000 u_{i-1} + 0.044800 u_{i-2} - \\
 & - 0.052480 u_{i-3} - 0.029952 u_{i-4} - 0.001485 u_{i-5} + 0.005396 u_{i-6} + \\
 & + 0.002456 u_{i-7} - 0.000097 u_{i-8} - 0.000530 u_{i-9} - 0.000193 u_{i-10} + \\
 & + 0.000029 u_{i-11} + \dots
 \end{aligned}$$

Thus we obtained an equation for the linear transformation of the original sequence

$$u_i^* = \sum_{p=-11}^{+1} k_p u_{i+p}. \tag{12}$$

To find the transfer function we use the well-known equation [2]

$$f(\omega_n) = \sum_{p=-11}^{+1} k_p e^{-ip\omega_n}. \tag{13}$$

Here $\omega_n = (\pi/N) n$, where N is the number of points at which the transfer function is determined and n takes the values $0, 1, 2, \dots, N$ (we took $N = 9$ in the calculation of $f(\omega_n)$).

In agreement with (13), we calculate the real and imaginary parts of the transfer function, which have the form:

$$\begin{aligned}
 R(\omega_n) &= \sum_{p=-11}^{+1} k_p \cos p\omega_n, \\
 I(\omega_n) &= \sum_{p=-11}^{+1} k_p \sin p\omega_n.
 \end{aligned}$$

From this we obtain the amplitude and phase:

$$\begin{aligned}
 |f(\omega_n)| &= \sqrt{R^2(\omega_n) + I^2(\omega_n)}, \\
 \varphi(\omega_n) &= \arctg \frac{I(\omega_n)}{R(\omega_n)}.
 \end{aligned}$$

The values of $|f(\omega)|$ and $\varphi(\omega_n)$ are presented in Table 1 for certain ω_n or the periods $T_n = 2\pi/\omega_n$ corresponding to them. In addition, the values of $|f(\omega_n)|$ are given graphically in Figure 1.

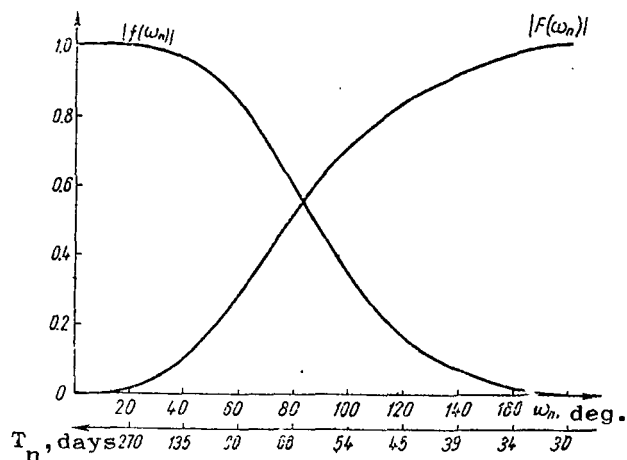


Figure 1.

smoothed curve undetected oscillations with periods not exceeding 35 days. Oscillations with periods of $35 \text{ days} < T_n < 270 \text{ days}$ occur both in the deviations and in the smoothed curve. Investigating these oscillations based on the deviations from the smoothed curve, it is necessary to calculate the amplitudes and phases of the harmonics of the original sequence from the equations:

$$\begin{aligned} A_d(\omega_n) &= A(\omega_n) |F(\omega_n)|, \\ \beta(\omega_n) &= \beta_d(\omega_n) + \varphi_d(\omega_n), \end{aligned} \quad (14)$$

TABLE

ω_n , De- gree	T_n , Days	$ ft(\omega_n) $	$\varphi(\omega_n)$, Degrees	$ F(\omega_n) $
0	∞	1.000	— 0.0	0.000
20	270	0.998	— 0.6	0.010
40	135	0.967	— 4.7	0.087
60	90	0.832	—13.9	0.278
80	68	0.579	—23.7	0.524
100	54	0.332	—27.6	0.723
120	45	0.165	—25.3	0.854
140	39	0.066	—18.6	0.938
160	34	0.020	— 7.6	0.980
180	30	0.000	— 0.0	1.000

where

$A(\omega_n), \beta(\omega_n)$ are the amplitude and phase of the n -th harmonic of the observed sequence of corrections;

$A_d(\omega_n), \beta_d(\omega_n)$ are the amplitude and phase of the same harmonic of the sequence of deviations from the smoothed curve; and

$|F(\omega_n)|, \varphi_d(\omega_n)$ are the amplitude and phase of the transfer function of the operator of the deviations.

The quantities $|F(\omega_n)|$ and $\varphi_d(\omega_n)$ are associated with the amplitude and phase of the transfer function of the smoothing operator, $|f(\omega_n)|$ and $\varphi(\omega_n)$ respectively, by the relations

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$$\begin{aligned} |F(\omega_n)| &= \sqrt{1 - 2|f(\omega_n)| \cos \varphi(\omega_n) + |f(\omega_n)|^2}, \\ \varphi_d(\omega_n) &= \frac{|f(\omega_n)| \sin \varphi(\omega_n)}{1 - |f(\omega_n)| \cos \varphi(\omega_n)}. \end{aligned} \quad (15)$$

In the particular case where the smoothing operator is symmetrical, i.e., $k_p = k_{-p}$, there occurs an equality

$$\cos \varphi(\omega_n) = 1, \quad \sin \varphi(\omega_n) = 0,$$

consequently,

$$\begin{aligned} |F(\omega_n)| &= 1 - |f(\omega_n)|, \\ \varphi_d(\omega_n) &= 0. \end{aligned}$$

The parabolic smoothing operator found by us is not symmetrical; therefore, it is necessary for the calculation of $|F(\omega_n)|$ and $\varphi_d(\omega_n)$ to use Eq. (15). The values of $|F(\omega_n)|$ are given in Table 1, and they are also presented graphically in Figure 1.

Finally, the long-period oscillations ($T_n > 270$ days) are transferred completely to the curve of the smoothed clock corrections.

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DETERMINATION OF THE CONSTANTS OF ROTATION
OF THE MOON TAKING INTO ACCOUNT THE ERRORS
IN THE COORDINATES IN THE CRATER MÖSTING A

V.S. Kislyuk¹

ABSTRACT: The author presents the values of the rotational constants of the moon, the errors in the coordinates of the crater Mösting A with respect to the system of limb craters being taken into account.

The found values of I and f do not practically differ from those presented in [1].

In the paper [1] the values of the constants of rotation of the moon

$$I = 1^{\circ}32'47'' \pm 24'', \\ f = 0.82 \pm 0.05,$$

are presented, which were obtained from the distances of 18 edge craters from the central crater Mösting A measured on 26 plates. These results agree well with the values obtained by A.A. Gorynya [2] using the method of the position angles of a large number of observations, which confirms the satisfactory representation of our comparatively small series of observations. However, in the paper [2], the problem of the effect of the errors of the coordinates of the craters used on the results obtained is not expounded. /66

In the present article we discuss the problem of the effect of errors in the coordinates of the crater Mösting A with respect to the system provided by edge craters on the determination of the constants of rotation of the moon. For this it is necessary to insert into the equations of condition, unknown corrections for the coordinates of the crater Mösting A in addition to the unknown corrections to the accepted initial values of I_0 and f_0 .

It is easy to show that

$$s_0 - s_c = \begin{Bmatrix} \sin p \\ \cos p \end{Bmatrix} \begin{pmatrix} \begin{pmatrix} A & A' \\ B & B' \\ C & C' \\ D & D' \\ E & E' \end{pmatrix} \begin{pmatrix} d\xi \\ d\eta \\ d\zeta \\ d\iota \\ df \end{pmatrix} \end{pmatrix} \quad (1)$$

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where

p is the position angle of an edge crater;

$d\xi$, $d\eta$, $d\zeta$, dI , df are the unknown corrections sought to the initial coordinates of the crater Mösting A and the values of I_0 and f_0 .

It is easy to show that

$$\begin{pmatrix} A & A' \\ B & B' \\ C & C' \end{pmatrix} = \begin{pmatrix} -A_{22} - A_{23} \\ -A_{32} - A_{33} \\ -A_{12} - A_{13} \end{pmatrix}, \quad (2)$$

where

A_{ij} are the elements of the well known Cracovian of T. Banakhevich.

The expressions for the coefficients D , D' , C , C' are obtained in the paper [1], and we will not present them here.

The following system of normal equations (the very same measurements as in paper [1]), are set up for 275 conditional equations:

dI'	df'	$d\xi'$	$d\eta'$	$d\zeta'$	l
120.921	3.468	3.045	-54.026	-7.528	1.898
	6.500	-5.630	-0.308	0.120	0.618
		125.937	-6.451	-2.293	1.791
			147.833	6.551	3.108
				1.282	-0.268

The unknown corrections are connected with the unknowns found from a solution of the system in the following manner:

$$d\xi = -\frac{d\xi'}{10}, \quad d\eta = \frac{d\eta'}{10}, \quad d\zeta = -\frac{d\zeta'}{10}, \quad dI = \frac{dI'}{100} \cdot \rho'', \quad df = df'. \quad (3)$$

They proved to be equal ($d\xi$, $d\eta$, $d\zeta$ are in units of $10^{-5}R_q$) to

$$\begin{aligned} d\xi &= -0.00006 \pm 0.00001, \\ d\eta &= +0.00015 \pm 0.00001, \\ d\zeta &= +0.00132 \pm 0.00059, \\ dI &= +17'' \pm 30'', \\ df &= +0.11 \pm 0.05. \end{aligned} \quad (4)$$

If one takes into account the fact that Gain's values were taken as the initial I and f , we obtain finally:

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$$\begin{aligned} I &= 1^{\circ}32'37'' \pm 30'', \\ f &= 0.84 \pm 0.05. \end{aligned} \tag{5}$$

It is obvious from this that the values sought are practically unchanged.

The problem of the effect of the errors of the coordinates of the edge craters remains open for the present.

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DETERMINATION OF CERTAIN INSTRUMENTAL CHARACTERISTICS
OF THE ZTL-180 ZENITH-TELESCOPE OF THE POLTAVA OBSERVATORY

N.A. Popov, N.I. Panchenko and A.T. Tsapova¹

ABSTRACT: The article deals with the basic instrumental characteristics of the Poltava Observatory ZTL-180 zenith-telescope - values of divisions of the Talcott levels, the scale value of the micrometer screw, the distances between horizontal hatching on the micrometer glass plate and others.

As a result of the investigation, the authors came to a conclusion that the temperature coefficient of these distances is very small.

The ZTL-180 zenith-telescope, one of a series of Soviet zenith-telescopes with an objective aperture of 180 mm [1], was acquired by the Poltava Gravitometric Observatory with the purpose of having a sufficiently powerful instrument with a wide field of view for carrying out observations of the variation in latitude. It was primarily intended to use this instrument for observations of latitude according to a zenith program, including observations of the bright zenith stars α Persei and η Ursae Majoris which have been carried out at Poltava since 1939. (Further observations with the Zeiss zenith-telescope have become difficult because of insufficient width of the field.)

The ZTL-180 is mounted in a specially constructed pavilion to the north from the Zeiss zenith-telescope pavilion at a distance of 56 m. The construction of the pavilion of the ZTL-180 is unique and successful. The pavilion is described in detail by the man who constructed it - chief mechanic of the observatory B.A. Sokoloviy.

The pavilion is of square construction with an area of 5.6 m x 5.6 m mounted on a masonry foundation. Its walls consist of a metal framework covered outside by corrugated galvanized iron and inside by wooden panels. The region between the inner and outer covering is filled with rock wool. The general appearance of the pavilion is shown in Figure 1.

Initially, the roof sections were moved with the help of electric motors directly by means of a system of flexible cables, which very often during the

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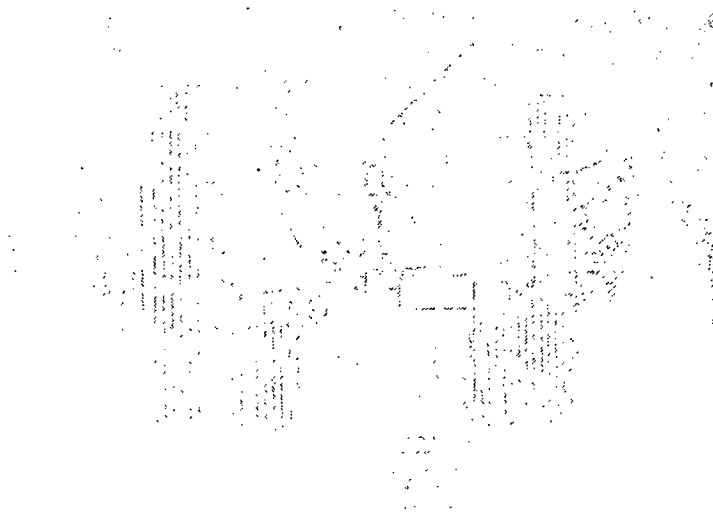


Figure 1. View of the pavilion.

cold time of the year operated with irregularities. Therefore, the mechanism ^{/68} was significantly changed and the cables were eliminated. At the present time, the sections are moved by chains with coarse spacing, to which forces are transmitted from gears placed on two special shafts in the eastern and western parts of the pavilion. Such a system has proved to be more reliable in practice and provided normal operation of the roof mechanism in the winter.

The instrument was mounted on the pier in May 1966. Unfortunately, fine adjustments and various finishing touches on separate parts of the instrument did not permit the completion of all the investigations in the summer and fall of 1966.

During preparation of the instrument for the observations a series of investigations was carried out of the fundamental instrumental characteristics. The results of these investigations are expounded in the present article.

Investigation of the Levels

The investigation of the Talcott levels was carried out on a testing machine which was mounted on a masonry pier in the pavilion of the APM-10 transit instrument. During January-July 1965 each of the levels was investigated 10 times at various values of the temperature. This work was carried out by the observers V.G. Sorokin and R.I. Popova. The limits within which the temperature varied during the observations were from +27° to -20°.

On the basis of the results of these measurements, the following values were obtained for the scale divisions of the levels and the temperature coefficients:

$$\begin{array}{ll} \text{Level No. 1 (lower)} & \tau = 1''.047 - 0''.0003 t, \\ & \pm 4 \quad \pm 3 \\ \text{Level No. 2 (upper)} & \tau = 0''.992 + 0''.0011 t. \\ & \pm 8 \quad \pm 4 \end{array}$$

Regardless of the comparatively large range of the temperature variations under which the levels were investigated, the temperature coefficients were not determined with certainty. This is possibly explained by their very small size. Consequently, for the time being there is no basis to take them into account in processing observations.

Determination of the Distances of the Side Threads (Verticals) from the Central One

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The ocular micrometer of the ZTL-180 has instead of a grid of spider threads a glass plate on which is placed a grid of lines. Instead of unmovable threads in the field of view, there are visible 13 lines which are paralld to the axis of the screw. The distance of each of these side vertical lines f from the central line was determined, as usual, from observations of stars, recording the instants of the transits of the stars with the help of a recording chronograph. These distances are as follows:

Line number	1	2	3	4	5	6	7	8	9	10	11	12	13
f	27 ^s .08	21.26	15.50	9.64	6.44	3.20	0.00	3.28	6.46	9.64	15.46	21.30	27.12

By convention that line which first intersects the image of the star at the "east" position of the instrument's tube is taken as the first line.

It is necessary to note that the distances f of the micrometer of our instrument are almost equal in accuracy to the distances of these same lines in the other micrometers of the ZTL-180 and also possess symmetry of the side lines relative to the middle line.

Investigation of the Usual and Periodic Errors of the Screw of the Ocular Micrometer

The usual errors of the screw of a micrometer were determined with the help of a special microscope with a scale from the Zeiss zenith-telescope. This microscope is so constructed that one can establish the errors of the screw, without removing it from the instrument [2]. The interval between

two bisectors in the field of view of the scale microscope was measured and was found equal to two screw revolutions of the micrometer of the ZTL-180.

In all, 20 series of measurements were made in two stages, with 10 series of measurements in each stage. After this, we compared the two series of corrections for the usual errors of a micrometer screw, and each series was obtained as the average of the 10 series of measurements. The agreement between both series was good. Therefore, the values of the corrections were finally calculated on the basis of all 20 series. The values of the corrections are presented in Figure 2.

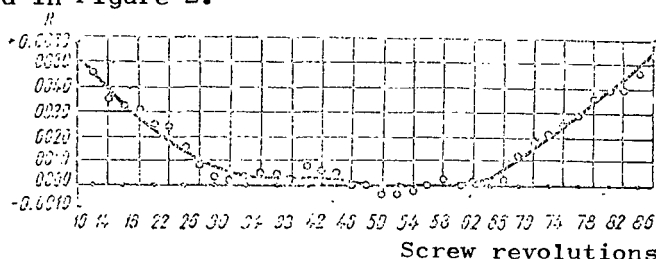


Figure 2. Corrections for the usual errors of the micrometer screw.

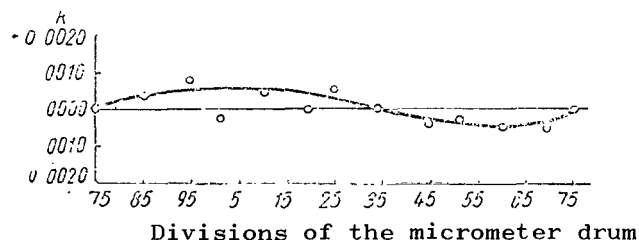


Figure 3. Corrections for the periodic errors of the micrometer screw.

As is evident from Figure 2, the corrections for the usual errors reach 0.0005^R only at the edges of the part of the screw investigated by us (10-86 revolutions), and in the interval between 42 and 64 revolutions they are practically equal to zero. /70

The values of the corrections for the periodic errors of the screw are presented in Figure 3. The investigations were carried out in the middle part of the screw at 48-49 revolutions. The size of the corrections does not exceed 0.0005^R .

The results of the investigations of the usual and periodic errors confirm the high quality of the screw of the ocular micrometer. This situation has already been remarked upon in a series of projects carried out at other ob-

servatories with the ZTL-180 instrument [3].

Determination of the Temperature Coefficient and Distances
between the Horizontal Lines of the Glass Plate of the
Poltava ZTL-180's Ocular Micrometer

Latitude observations of zenith stars by the Talcott method can be completely successful only when the zenith distances of the stars are small. However, it is very difficult to make up a suitable program from zenith stars under this condition, since there are very few such stars in the narrow zone of the sky accessible for observations with the zenith-telescope. Moreover, they recede from the zenith because of the effect of precession and pass out of this zone comparatively soon if one does not consider those which culminate around 6 and 18 hours local sidereal time.

In order that the zenith program be the most permanent and suitable for observations (from the point of view of the possibility of observing one and the same zenith star at two tube positions), it is necessary to widen the limits of measurements of the differences of the zenith distances by the micrometer screw. One can achieve this if the supplementary horizontal threads are stretched on the moveable frame of the micrometer [4] or there is attached on the frame a transparent plate with engraved lines. Then even for small numbers of revolutions of the micrometer screw, it is possible to carry out observations using not one but two different lines. In this way, it becomes possible to observe stars which cross the meridian at considerably greater distances from the zenith. The selection of the stars is significantly facilitated since a wider zone in declination is selected.

On normal zenith-telescopes one can widen the zone of measurements with a micrometer by the method of setting the supplementary threads or the cuts of the lines on the glass plate a maximum of up to 30'. Consequently, it is possible to observe only those zenith stars which are distant by no more than 15' from the zenith.

A remarkable feature of the ZTL-180 consists in the wide angle of its objective; the size of the visible field of view reaches $1^{\circ}45'$, and the working field reaches $1^{\circ}30'$. Thanks to this, observations of the zenith program by the ZTL-180 can be carried out without changing stars for more than 150 years. A program of Talcott star pairs can be made up for a still more

lengthy term of observations.

The width of the field and the series of lines (13 in all), which are uniformly distributed along the entire length of the glass plate, permit the determination of the value R of a screw revolution of the ocular micrometer from observations of wide scale pairs, by which a very high accuracy of the obtained value for the scale of the measurements is achieved [5, 6, 7, 8].

The enumerated features of the instrument permit its use in solving a more extensive range of problems than it is possible to solve with instruments of the previous type of construction. However, the distances between the lines L_i should be carefully determined and thoroughly investigated for their temperature variations. Thus, it is necessary to derive the distances from series of separate measurements obtained at various values of the temperature. /71

The measurements of the distances of the side vertical lines from the central (which are oriented parallel to the meridian) were carried out by us from observations of star transits with an accuracy of ± 0.02 , but the distances between the horizontal lines (which are perpendicular to the meridian) must be determined with a far greater accuracy. A special microscope having a scale with engraved bisectors, which we have already referred to, was used for this purpose. The measurements were carried out during the daytime on the same instrument using the method which L.P. Basurmanova applied [9].

The distances between two neighboring lines amounts to almost 20 screw revolutions, and the screw had 100 revolutions in all. Therefore, at a single setting of the bisector it is possible to measure by revolutions of the micrometer screw only four distances if the extreme revolutions from the 1st to the 10th and beyond the 90th are not used. Thus, it is necessary to shift the platen with the microscope screwed into the ocular aperture of the micrometer twice in order to measure the 12 distances from the 1st to the 13th line.

In the first method we measured the distances between the 1st and 2nd, 2nd and 3rd, 3rd and 4th, 4th and 5th lines - forward motion. Then these measurements were repeated in the reverse direction, returning to the 1st line and the 10th revolution on the micrometer barrel. Then, after the first resetting of the platen, during which the bisector was shifted from the 1st

to the 5th line, similar measurements were carried out between the 5th and 9th lines. Finally, after the second resetting the distances between the 9th and the 13th lines were measured.

The measurements of the distances L_i were made only in the middle of the field of view, i.e., along the 7th vertical line. Such measurements must be made in the upper and lower parts of the field in order to determine deviations of the lines from parallelness.

During the period from August 11 until December 22, 1967, the quantity L_i was measured nine times at various values of the temperature. The results are expressed in revolutions of the micrometer screw and also corrected for its normal errors (Table 1).

TABLE 1.

Line No.	Temperature, °C								
	+23	+28	+16	+8	-4	0	-16	-15	-18
1-2	19.9930	19.9984	20.0004	19.9991	20.0000	20.0000	20.0007	20.0022	20.0015
2-3	19.9876	19.9884	19.9835	19.9924	19.9905	19.9919	19.9903	19.9893	19.9905
3-4	20.0112	20.0096	20.0107	20.0089	20.0086	20.0099	20.0101	20.0094	20.0101
4-5	19.9963	19.9971	19.9960	19.9944	19.9943	19.9929	19.9931	19.9948	19.9943
5-6	20.0015	20.0023	20.0043	20.0033	20.0040	20.0037	20.0037	20.0055	20.0036
6-7	19.9827	19.9831	19.9829	19.9841	19.9831	19.9838	19.9842	19.9820	19.9834
7-8	20.0133	20.0139	20.0128	20.0137	20.0138	20.0124	20.0131	20.0134	20.0149
8-9	19.9923	19.9895	19.9907	19.9897	19.9913	19.9894	19.9882	19.9871	19.9874
9-10	20.0058	20.0074	20.0074	20.0086	20.0084	20.0065	20.0076	20.0075	20.0082
10-11	19.9791	19.9801	19.9797	19.9813	19.9829	19.9835	19.9804	19.9795	19.9811
11-12	20.0360	20.0250	20.0265	20.0266	20.0246	20.0256	20.0272	20.0253	20.0246
12-13	19.9924	19.9956	19.9912	19.9910	19.9900	19.9918	19.9883	19.9904	19.9916
1-13	239.9862	239.9904	239.9921	239.9931	239.9915	239.9914	239.9869	239.9864	239.991

The combination of the measurements of the plate and the micrometer screw as a function of the temperature affects the distances between the lines; the larger the distances, the greater is the difference in the temperature coefficients of the linear expansions of the type of glass and steel selected.

There is in general no standard designation for this "combined" temperature coefficient. We will denote this quantity by κ , since we already adopted such a designation earlier when the temperature variation between the supplementary movable threads on the Zeiss zenith-telescope were investigated [10].

The effect of the temperature variation of the length is determined more exactly in this case if it is possible to take as the unit of measurement a larger distance on the plate. Therefore it is advisable to determine the

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quantity L for the distance between the 1st and 13th lines. This distance is found by the method of adding up the columns of Table 1 of all the values of L_i which are measured at various temperatures from $+28^\circ$ to -18°C . On the basis of the data of Table 1, nine conditional equations of the form $L_t = L_0 + \kappa t$ were formulated. The solution by the method of least squares leads to the following expression:

$$L_t = 239.9900 - 0.00005t.$$

$\pm 9 \qquad \qquad \pm 5$

As is evident, the distance, which is equal to 240 screw revolutions, changes very little upon a temperature change of 1° , and the size of this change lies within the limits of error of the measurements. If the value of the temperature coefficient is formally calculated, relative to the distance of one revolution of the micrometer barrel, then it amounts to 0.0000002 (in screw revolutions) or in angular measure, 0.0000044. If this value were real, then one could neglect it in all those cases where the measurement differences of the zenith distances did not exceed $35'$ with temperature oscillations within the limits of 50° .

Using the data presented in Table 1, we obtain the final values of the distances between each pair of horizontal lines of the micrometer grid (Table 2).

TABLE 2.

Line No.	Distance	Line No.	Distance
1—2	20.0000 ± 5	7—8	20.0135 ± 2
2—3	19.9900 ± 5	8—9	19.9895 ± 6
3—4	20.0098 ± 3	9—10	20.0075 ± 3
4—5	19.9948 ± 5	10—11	19.9808 ± 5
5—6	20.0035 ± 4	11—12	20.0257 ± 3
6—7	19.9833 ± 2	12—13	19.9914 ± 6

Determination of the Value of a Screw Revolution of the Ocular Micrometer

To determine a preliminary value R , scale pairs were set up as long as they were not very wide; the component of the difference of zenith distances in the pairs does not exceed 120 revolutions. Under these conditions, as our investigations have shown, one can neglect the temperature variations of

the differences between the movable lines. The declinations and proper motions of the stars were taken from the Yale catalogue [11].

Calculations of R were carried out in the following manner. The measured difference in zenith distances of the stars is represented by the expression

$$R[l + (M_1 - M_2)] + \Delta\rho + k_1 - k_2, \quad (1)$$

where

l is the distance between the two lines by which sightings were made so that one was on the first and the other was on the second star.

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These values were taken from Table 2. For example, if the sighting was carried out with the 5th and 9th lines, then in such a case $l = 79.9898$ screw revolutions. The correction for differential refraction $\Delta\rho$ was calculated from the equation $\Delta\rho = \alpha\beta\gamma \sec^2 z \Delta z$, where z is the zenith distance of the center of the scale pair and Δz is the difference in zenith differences of the components of the pair, expressed in minutes of arc. The size of $\alpha\beta\gamma \sin 1'$ for various values of the temperature and the atmospheric pressure were taken from the table presented in Yumi's article [12]. Corrections for the curvature of the parallel for the first and second stars are denoted by k_1 and k_2 , respectively.

Equating the measured distance in zenith distances (1) with the calculated, i.e., the differences in the declinations of the stars $\delta_1 - \delta_2$ reduced to the epoch of the observations, we obtain the following relation for the derivation of the value of a screw revolution:

$$R = \frac{\delta_1 - \delta_2 - \Delta\rho + k_2 - k_1}{l + M_1 - M_2}$$

On the basis of the results of the observations of scale pairs, which were carried out during December 1967 - January 1968, we obtained a preliminary value R equal to $21''.8791 \pm 0''.0009$. Some characteristics of the stars which were joined as pairs are presented in Table 3, along with the values R calculated from the observations of separate scale pairs. The numbers of the stars are shown according to the Yale catalogue and the number of observations of a given pair is denoted by n .

Disregarding the comparatively small number of observations of each pair, the agreement between the separate values of R , as is evident from Table 3,

TABLE 3

No. of star	<i>m</i>	α_{1967}	δ_{1967}	<i>n</i>	<i>R</i>
425	7.2	0 ^h 55 ^m	52°04'		
472	7.6	1 01	52 40	3	21".8736
713	7.6	35	51 35		
746	7.7	37	52 06	2	.8834
846	5.9	49	51 46		
868	6.2	51	51 19	4	.8832
914	7.9	56	52 55		
944	7.8	2 00	53 28	1	.8762
1021	6.6	11	52 54		
1045	6.3	13	52 21	6	.8809
1141	7.2	26	52 24		
1203	7.1	35	51 49	3	.8809
1281	7.9	46	51 25		
1297	7.2	48	52 09	6	.8762

is sufficiently good. This is evidently explained by the adequacy of the method of determining *R* from wide scale pairs, which has already been repeatedly mentioned in the literature [5, 7, 8].

Subsequently, we must derive the value of *R* from a considerably larger amount of observational material, which it is necessary to carry out under large temperature variations in order to determine the temperature coefficients of the value of a screw revolution of the ocular micrometer.

We note in conclusion that, as our investigations have shown, the basic instrumental characteristics of the ZTL-180 zenith-telescope of the Poltava Observatory are very similar to the corresponding characteristics of the zenith-telescopes at Pulkovo, Moscow, Blagoveshchenska, and other places.

In particular, we should note the very important fact that the factory succeeded in attaining good temperature compensation in the preparation of the micrometer screw and the glass plate. This has great significance for the observations of zenith stars and the determination of *R* from wide scale pairs.

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THEORY OF THE DETERMINATION OF TUBE FLEXURE
WITH THE USE OF AUTOCOLLIMATION

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ABSTRACT: The author considers theoretically the dependence between the angle of rotation of the flat mirror, rigidly fastened to the objective casing, and the bend angle of the objective end of the tube.

On the basis of this dependence, the autocollimation angle is represented by a linear function of the bend angles of both ends of the tube. It enables the differential flexure of the tube to be determined by means of measuring the autocollimation angle by a flat mirror fastened before the objective glass and the bend angle of the eye-piece of the tube by a collimator placed in the cube.

The author proposed the use for determining the tube flexure of the Vanshaff vertical circle of autocollimation from a flat mirror fastened on a mounting in front of the objective in combination with an independent determination of the bending of the ocular end of the tube with the help of a collimator mounted on a cube. It is shown in the paper [1] in particular that this method could become completely faultless if it were possible to determine the deformational state of the tube or establish the relation between the angle of deflection of the mirror, which is rigidly fastened to the objective mounting, and the bending angle of the objective end of the tube at various zenith distances. A solution of the second version of this problem is presented in this paper.

It is evident that the two halves of the tube of any meridian instrument can be represented in the form of two arms with an inset in the central part of the tube or in a cube and loaded at the free end of one arm by the weight of the objective, and at the end of the other arm by the weight of the micrometer. The actual weight of the tube should be considered as a uniformly distributed load, and the effect of the weight of the objective and the micrometer should be considered as concentrated. /75

We discuss now the optical layout of the method. The shape of the tube is presented in Figure 1 by the shape of its central line A'mOnB'. O is the

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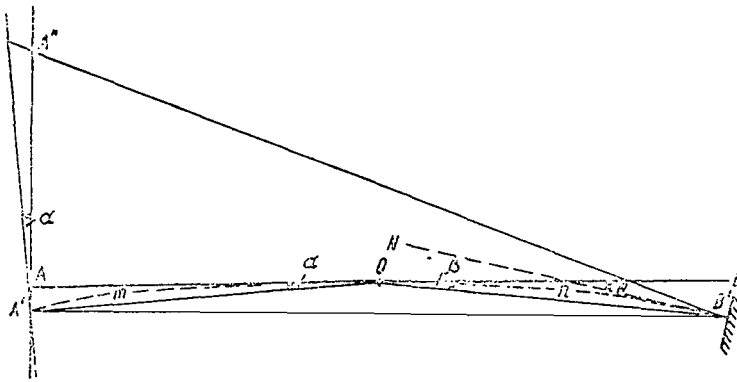


Figure 1. Optical layout of the autocollimation method.

center of the tube or cube - the point which divides the segment AB into two equal halves. AB is the axis of symmetry of the ideal undeformed tube, which is assumed to coincide with the sighting axis of the collimator which is mounted inside the cube for the determination of the bending of the ocular end of the tube. A' and A'' are the center of the cross wires and its autocollimated image, respectively. B' is the second main point of the objective. $\alpha = \angle AOA'$ and $\beta = \angle BOB'$ are, respectively, the bending angles of the ocular and objective ends. NB' is the normal to the mirror which is mounted in front of the objective and rigidly fastened to its mounting. θ is the angle formed by the normal NB' with the AB axis. $\gamma = \angle A'B'A''$ is the autocollimation angle measured as the ratio of the distance between the direct and reflected images of the cross wires to the focal length of the objective.

As follows from Figure 1, the angle between the normal NB' and the sighting line A'B' is equal to

$$\theta + \frac{\alpha - \beta}{2}.$$

Consequently, the angle γ is equal to the doubled value of this quantity

$$\gamma = \alpha - \beta + 2\theta \quad (1)$$

In the derivation of this equation we neglected errors of the second order which arise because of the rotation of the focal plane by an angle α . This angle does not exceed 15-20" in normal cases.

Thus, if the quantities α and γ become known quantities from the measurements, then knowledge of the angle θ is necessary for determining the size of

the differential flexure

$$\Delta\zeta = \frac{1}{2}(\beta - \alpha) \quad (2)$$

The angle Θ is the angle formed by the tangent to the central line of the tube at the point of attachment of the mirror with the AB axis. It is possible to express it in terms of the angle β if the equation of the deformed central line of the objective part of the tube is known.

The tube of the Vanshaff vertical circle consists of two identical halves /76 of cylindrical shape, molded in a cube, which presents a thick-walled cylindrical coupling with the thickness of the walls, significantly exceeding the thickness of the walls of each of the cylinders. The free ends of each half of the tube are fastened with supplementary rings and are terminated by completely identical ring flanges. On the basis of such construction, and making an assumption about the undeformability of a cube and the objective mounting together with part of the tube, which is reinforced by rings, it is possible to present the following static model of the tube (Figure 2).

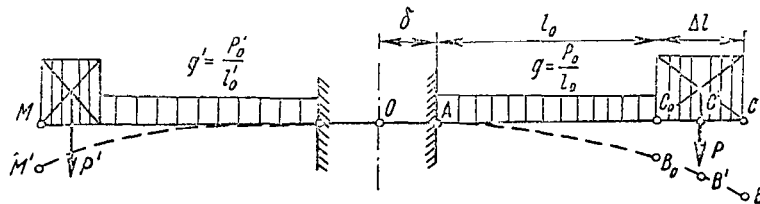


Figure 2. Static model of the tube.

Here AC_0 is the deformed segment of the objective part of the tube of uniform cross section with a moment of inertia of the transverse cross section I . OA and C_0C are the undeformed parts of the tube - the cube and the objective mounting together with the part of the tube reinforced by rings and a flange, which it is also possible to consider as undeformed and discuss as part of the mounting. A is the point of intersection of the central line with the front plane of the cube, which it is possible to consider as the point of attachment of the arm. C_0 is the point of connection of the tube with the mounting or the last point of the deformed part of the tube. P is the resultant weight of the undeformed parts of the tube located on its objective end - the objective, its mounting, and the strengthening rings and flange. C' is the point of application of the resultant P . $P_0 = ql_0$ is the weight of the deformed uniform part of the objective end of the tube (l_0 is the

length of this part and q is the strength of a uniformly distributed load).

The differential equation of the central (elastic) line of the arm in the rectangular system of coordinates $yC'z$ (Figure 3) has the form

$$EI \frac{d^2 y}{dz^2} = M(z), \quad (3)$$

where

E is the modulus of elasticity of the arm's material and $M(z)$ is the bending moment.

As a result of integrating Eq. (3) the 77 angle of rotation of the cross section and the bending yield the expressions

$$\theta(z) = \frac{1}{EI} \left[\int M(z) dz + D_1 \right] \quad (4)$$

$$y(z) = \frac{1}{EI} \left[\int dz \int M(z) dz + D_1 z + D_2 \right]. \quad (5)$$

The constants of integration and Eqs. (4) and (5) are determined from the boundary conditions.

Figure 3. Auxiliary sketch for the derivation of the equation of the elastic line.

We now find the quantities θ_1 and y_1 at the point C_0 under the influence of a concentrated force applied at the point C' and θ_2 , y_2 for the same point under the influence of a uniformly distributed force P_0 . To find the first pair of values θ_1 and y_1 we take the origin of the rectangular system at the point C' . The bending moment from the concentrated force P in this case is

$$M_1(z) = -Pz.$$

Eq. (3) is written in the form

$$EI \frac{d^2 y}{dz^2} = -Pz, \quad (6)$$

and the Eqs. (4) and (5) respectively are

$$\theta(z) = \frac{1}{EI} \left[-\frac{Pz^2}{2} + D_1 \right] \quad (7)$$

and

$$y(z) = \frac{1}{EI} \left[-\frac{Pz^3}{6} + D_1 z + D_2 \right]. \quad (8)$$

The points of clamping of the arm, the angle of rotation, and the bending are equal to zero for the point A . Therefore, the boundary conditions are written in the form

$$\vartheta(l_0 + \Delta l_0) = 0,$$

$$y(l_0 + \Delta l_0) = 0.$$

After their substitution into (7) and (8) we find:

$$D_1 = \frac{P}{2} (l_0 + \Delta l_0)^2,$$

$$D_2 = -\frac{P}{3} (l_0 + \Delta l_0)^3.$$

We obtain the values Θ_1 and y_1 sought after substitution into (7) and (8) of the value $z = \Delta l_0$ - the coordinates of the point C_0 in the accepted system of coordinates and the values D_1 and D_2 found:

$$\Theta_1 = \frac{Pl_0}{2EI} (l_0 + 2\Delta l_0), \quad (9)$$

$$y_1 = -\frac{Pl_0^2}{6EI} (2l_0 + 3\Delta l_0). \quad (10)$$

To detect the effect on the point C_0 of a uniformly distributed load with a strength /78

$$q = \frac{P_0}{l_0}$$

we discuss a similar system of coordinates with the origin at the point C_0 .

The bending moment in the z cross section is

$$M(z) = -q \frac{z^2}{2},$$

and Eq. (3) is written thus:

$$EI \frac{d^2 y}{dz^2} = -q \frac{z^2}{2}. \quad (11)$$

The boundary conditions for the same point A which has in the new system the coordinate $z = l_0$ are similarly

$$\Theta(l_0) = 0,$$

$$y(l_0) = 0.$$

The angle of rotation and the bending at the point C_0 with coordinate $z = 0$ after the substitution into the first and second integrals of Eq. (11) of the corresponding values of the constants of integration and the values $z = 0$ are found to be the following:

$$\Theta_2 = \frac{P_0 l_0^2}{6EI}, \quad (12)$$

$$y_2 = -\frac{P_0 l_0^3}{8EI}. \quad (13)$$

Thus the total angle of rotation C_0 under the effect of concentrated and

uniformly distributed loads is found according to the Eqs. (9) and (12) to be

$$\Theta = \frac{1}{EI} \left[\frac{Pl_0}{2} (l_0 + 2\Delta l_0) + \frac{P_0 l_0^2}{6} \right]. \quad (14)$$

The total bending at the same point, according to (10) and (13), is

$$y = -\frac{1}{EI} \left[\frac{Pl_0^2}{6} (2l_0 + 3\Delta l_0) + \frac{P_0 l_0^3}{8} \right] \quad (15)$$

Furthermore, noting from the triangle AC_0B_0 (Figure 4), that for sufficiently small β_0

$$-\frac{y}{l_0} = \beta_0,$$

the Eq. (15) is written as follows:

$$\beta_0 = \frac{1}{EI} \left[\frac{Pl_0}{6} (2l_0 + 3\Delta l_0) + \frac{P_0 l_0^2}{8} \right]. \quad (16)$$

Equating now the relations of the left and right parts of Eqs. (14) and (16), we find

$$\Theta = 4 \frac{3P(l_0 + 2\Delta l_0) + P_0 l_0}{4P(2l_0 + 3\Delta l_0) + 3P_0 l_0} \beta_0. \quad (17)$$

If the point of application of the resultant and also the 2nd main point of the objective would coincide with the point C_0 - the end of the deformed part of the tube, then the desired relation of the angles Θ and β would be represented by the expressions /79

$$\Theta = 4 \frac{3P(l_0 + 2\Delta l_0) + P_0 l_0}{8P + 3P_0} \beta. \quad (18)$$

Since, however, such a condition cannot be fulfilled by an actual instrument, then it is possible to discuss Eq. (18) only as an approximate form. The exact dependence should be found from Eq. (17) by replacing the angle β_0 by its expression in terms of the angle β . We make this transition in two steps, first going from the angle β_0 to the angle β' .

Let l_0 be the length of the deformed part of the tube and δ be half the length of the cube. If these quantities are known to us, then the relation between the angles β_0 and β' , as can easily be seen from the triangles AB_0C_0 and OB_0C_0 (see Figure 4), are represented by the expression

$$\beta_0 = \left(1 + \frac{\delta}{l_0} \right) \beta', \quad (19)$$

Substituting (25) into (20), we obtain

$$\Theta = k \frac{l + \Delta l}{l + k \Delta l} \beta. \quad (26)$$

Eq. (26) represents the desired relation between the angles Θ and β .

We now turn to Eq. (1) and insert in it the value of Θ from Eq. (26).

After rearrangements we find

$$\gamma = \alpha + k_0 \beta, \quad (27)$$

where

$$k_0 = \frac{k \left(2 + \frac{\Delta l}{l} \right) - 1}{1 + k \frac{\Delta l}{l}}. \quad (28)$$

Thus, it follows from Eq. (27) that the autocollimation angle γ is expressed by a linear function of the quantities α and β - the angles of bending of the ocular and objective ends of the tube.

The constant k_0 depends only on the geometrical dimensions and weight characteristics of the tube and, consequently, cannot change if the latter quantities remain unchanged. It also follows from Eqs. (21) and (28) that the quantity k_0 cannot change in the event of change of position of the tube in zenith distance.

Example. On the basis of the measurements made of the Vanshaff vertical circle it was found: $P = 175.20$ n; $P_0 = 70.58$ n; $l_0 = 814.0$ mm; $\Delta l_0 = 134.8$ mm; $\delta = 218.4$ mm; $l = 1032.4$ mm; and $\Delta l = 227.6$ mm. In calculating the weight P_0 the specific weight is taken as 8120 kg/m^3 , (brass tube), and in the calculation of the quantity Δl we took

$$l + \Delta l = \frac{1}{2} F,$$

where

F is the focal length of the objective.

On the basis of the data presented it was found that

$$k_0 = 2.3788,$$

and, consequently, Eq. (27) for the tube of the Vanshaff vertical circle takes the form

$$\gamma = \alpha + 2.3788 \beta. \quad (29)$$

Eq. (29) is obtained for the case of the absence of additional weights on the tube. Small additional weights hung on the tube for adjustment of the amount of differential flexure affects k_0 rather insignificantly. We present the

values of the quantity k_0 for the case of loading of the objective end of the tube by standard counterweights with weights of 10.80 and 27.53 N;

P	k_0
10.80	2.3771
27.53	2.3781

Bearing in mind the simplification made in the derivation of Eq. (27) and 81 also the fact that the original data were taken from approximate measurements, it is impossible to consider the value of k_0 found by such a method as the final one. The final value of the quantity k_0 should be taken from a comparison of the results of direct measurements of the quantities α and γ with the amount of flexure in the horizontal, which can be obtained with the help of horizontal collimators.

It is possible also to find the quantity k_0 by a direct measurement of the bending of the ends of the tube and the amount of autocollimation, approximately the same as was done by N. Hansson during an investigation of the flexure of the meridian circle of the Lund Observatory [2].

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PHOTOEXPOSURE METER FOR PHOTOGRAPHIC
RECORDING OF CIRCLE READINGS

A.M. Stafeyev and I.N. Nabokov¹

ABSTRACT: A photoelectrical apparatus is suggested to be used for choosing exposure during photographing the meridian circle limb. The authors made such an apparatus and called it photoexposimeter. A description of this apparatus and its design are presented.

The quality of the plates obtained, on which the accuracy of the circle readings essentially depend, is of particular significance for providing highly simplified observations of the declinations of stars in the case of photographic recording of the circle readings.

Particular attention should be paid during the preparation of photomicroscopes, to the system of illumination of the graduated circle, which should ensure obtaining bright lines with sharp edges and a dark background between lines on a fine-grained plate.

Since a circle reading is an average reading from 4 to 6 photomicroscopes, the quality of the images of the circle divisions on all the plates should be identical.

At the present time in all photographic systems of circle readings fixing the quality of the images of the lines of a graduated circle on the surface of the emulsion of the plate and also the determination of the intensity and uniformity of illumination is carried out either simply visually by means of a microphotoadapter or from the visible image on a diffuse screen mounted in the plane of the plate.

Thus, it is very difficult to determine at once the amount of exposure for a given intensity and uniformity of illumination. Therefore, these quantities are selected by obtaining several photographs for each adjustment. Much trouble and time accompanies this operation.

In addition, the difficulty of visual fixation includes yet the fact that the intensity of the illumination should be identical for all photo-

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microscopes.

Therefore, in 1967 we constructed and prepared a photoelectrical apparatus (photoexposure meter) which would rapidly and accurately record the intensity and nonuniformity of the illumination.

The photoexposure meter consists of a receiving instrument, an amplifier, and a recording device (photograph).

The receiving instrument consists of a plate holder and an FD-2 photodiode. The plate holder serves for the attachment and adjustment of the displacement of the FD-2 along the aperture of the frame. The light-sensitive slit of the FD-2 with a diameter of around 1 mm permits determining the nonuniformity of illumination over the entire field of the frame.

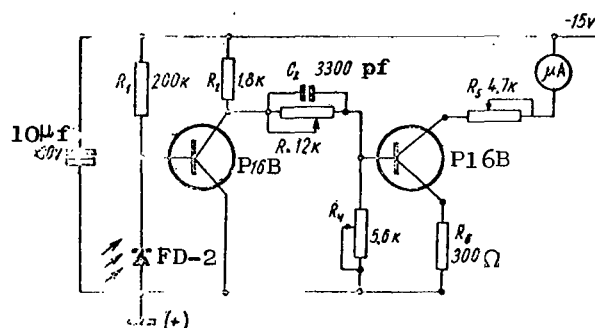


Figure 1. Photoexposure meter.

The amplifier part of the photoexposure meter is composed of two semiconductor triodes of the P16B type. A potentiometer R_5 and a recording apparatus - a microammeter - are placed at the output of the latter in the collector circuit.

The photoexposure meter operates in the following way. The light reflected from the graduated circle, passing through the optical system of the photomicroscope and the region of the plane of the plate, falls on the light-sensitive layer of the FD-2 photodiode, causing a photocurrent on it.

The current from the FD-2 enters the amplifier, and from it - the microammeter. The reading on the scale of the microammeter is proportional to the photocurrent which arises in the photoelement which, in its turn, is proportional to the intensity of the light falling on the photoelement.

For each setting of the FD-2 along the aperture of the frame, the intensity and nonuniformity of the illumination of the graduated circle are registered by the readings of the microammeter.

It is desirable to calibrate the adjusting resistor R_5 of the photoexposure meter for the setting for various types of plates. It is necessary to note the

readings of the microammeter and the amount of exposure. Furthermore, such calibration permits rapid adjustment of the system of illumination and acquisition of a plate of high quality.

The application of the photoexposure meter permits the saving of much time, lessens the effort, improves the quality of the plates, and increases the accuracy of the recording of circle readings.

INVESTIGATION OF THE PIVOTS OF THE MERIDIAN CIRCLE
OF THE ODESSA ASTRONOMICAL OBSERVATORY

M. Yu. Volyanskaya and A.M. Stafeyev¹

ABSTRACT: The article presents the results concerning the investigation of the pivots of the meridian circle installed at the Odessa Astronomical Observatory. The investigation was carried out by the authors in October 1967. This work was done in two positions of the instrument using the contact method by means of a lever with a probe and a photomicroscope instead of a micrometer. The irregularities of the pivot radii are found in $2:5$. The values of collimation corrections Δc^s are presented.

This work was undertaken in October 1967 in connection with the observations with the meridian circle of the right ascensions of eclipsing stars and also because from the time of the preceding investigation almost 30 years had passed. The contact method adopted by us was realized with the help of a lever with a probe.

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The contact method described by Philippot [1] was used for the investigation of the pivots of the Odessa meridian circle by B.V. Novopashennyy in 1937 and by B.V. Novopashennyy and A.S. Tsesyulevich in 1939. The investigations of 1937 showed that the pivots have irregularities of the order of 1μ . The processing of the 1939 observations was not completed. We utilized the lever which was used in 1937. A steel ball with a diameter of 3 mm served as the probe on the lever. The lever was altered so that the probe touched the operating cross-section of the pivot at a point distant by an angle of $\gamma = 22:5$ from the top. The first nine harmonics of decomposition of the curve of radius corrections Δv_i are determined more accurately at $\gamma = 22:5$ than at $\gamma = 0^\circ$, as is shown in [2] and [3], and also the effect of accidental irregularities is decreased. A glass plate with a grid of horizontal and vertical lines was attached to the measuring end of the lever. A photomicroscope [4], in which cross hairs - a fixed index - were mounted between the first and second objective, was used to record the displacements of the end of the lever with the grid of lines. At

1. Odessa Astronomical Observatory.

each setting of the instrument during an investigation of the pivots the fixed cross hairs and the lines changing their position were photographed. Because of a change in the ratio of the arm of the lever, an increase by 14.6 times was obtained in the displacement of the probe under the action of the pivot irregularities. A photo-optical system gave a 4-power magnification. Thus, in the event of a displacement of the probe by 1μ a selected thread was shifted by 58.4μ . An electrical control panel for the photographic system of recording circle readings was used. The time of exposure was 5 seconds and the rewinding of the film was 2 seconds. It was decided to find the irregularities of the operating cross section of the pivots at every $2:5 - 144$ radius corrections in all. Two observers participated in the measurements: one rotated the axis by a control wheel, setting the instrument by the finder's circle through $22:5$ and the other observer photographed and recorded the readings and notes. Measurements with rotations of the instrument's axis by $22:5$ from 0° to 360° and back entered into one series. Each pivot was investigated separately on each support. Four cycles of measurements were obtained with nine series each which begin from 0° , $2:5$; 5° , ..., 20° . The investigation was carried out over four days. The temperature in the pavilion stayed within the limits of $11-17^\circ\text{C}$. In the course of one series of measurements, which occupied 15 minutes, the temperature was practically unchanged. The pivots and the supports were not exposed to any changes, and the instrument was in operating condition. /84

1188 frames of film were obtained. The measurement was frequently carried out by two people on a measuring machine [4] with an accuracy of $\pm 0.5 \mu$. The mean square error of a single measurement according to internal agreement was equal to $\pm 0.0012 \mu$, and the mean square error of a single reading during the investigation of the pivots was equal to $\pm 0.02 \pm 0.03 \mu$. In the course of further reduction after consideration of the magnification scale in each series, a mean of the direct and reverse runs was calculated, then linear compensation was carried out and the mean for the series was computed:

$$v_0 = \frac{\sum_{i=1}^{16} v_i}{16}.$$

The deviation of each reading from the mean for the series appears to be the result of the total effect of the pivot irregularities at the three points of contact with both bearings of the support and with the measuring probe. Thus,

the displacement of the probe is $L_i = v_1 - v_0$. On the other hand,

$$L_i = \Delta r_i + 0.383 \Delta r_{i+112^\circ.5} + 0.923 \Delta r_{i+202^\circ.5}$$

$$i = 0^\circ, 22^\circ.5, \dots, 337^\circ.5,$$

where

Δr_i are the pivot errors, i.e., the deviations of the separate radii of the pivots from the mean of their value.

So we obtained a system of 16 equations with three unknowns. The distinctive features of the solution of such a system of equations are discussed in the papers [3] and [5]. For a single series of measurements it is necessary to solve by means of correlation of the 16 systems and take the arithmetic average of the solutions obtained. For the four cycles it is necessary to solve 576 systems. These calculations were carried on the "Ural-2" machine of the Odessa State University. As a control, the values of Δr_i were calculated partly from the equations for the arithmetic mean presented in [3]. But prior to the solution of the systems of equations the sinusoidal wave was excluded from the values L_i . The appearance of a sinusoidal term in the values L_i was noted in particular in the papers [5] and [6], whose authors consider it to be the result of the effect of the inclination of the axis or some other reasons. The exclusion of the sinusoidal variation from the values L_i was carried out by the exclusion of either a sinusoid whose parameters were found from only 16 values entering into a single series or a common sinusoid whose parameters were found from all 144 values of the cycle. We calculated the sinusoidal variation by both methods, but only sinusoids from each series were excluded, since the amplitude of the common sinusoid was $\pm 0.036 \mu$ in all, i.e., of the order of the accuracy of the measurements. Its effect on the general variation of the L_i was negligible; therefore the necessity of taking it into account vanished. The values obtained for L_i appeared as free terms in the system of equations from which the Δr_i were found.

Knowing the pivot irregularities Δr_i , we found the displacement of the center of each pivot [7]:

$$\Delta x = \frac{1}{\sqrt{2}} (\Delta r_{i+135^\circ} - \Delta r_{i-135^\circ}),$$

$$\Delta y = \frac{1}{\sqrt{2}} (\Delta r_{i+135^\circ} + \Delta r_{i-135^\circ}).$$

Having found similar quantities for the other pivot and forming the differences of the displacements of the centers of both pivots for identical positions of the instrument i , we obtained the shift of the horizontal axis Δi_i and Δk_i .

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The error in collimation was calculated from the equation:

$$\Delta c^s = \frac{\Delta i_i \cos z + \Delta k_i \sin z}{D \sin 1^s},$$

where

D is the length of the horizontal axis of the meridian circle and is equal to $958 \times 10^3 \mu$.

The values of the collimation corrections Δc^s for two positions of the instrument are presented in Figure 1.

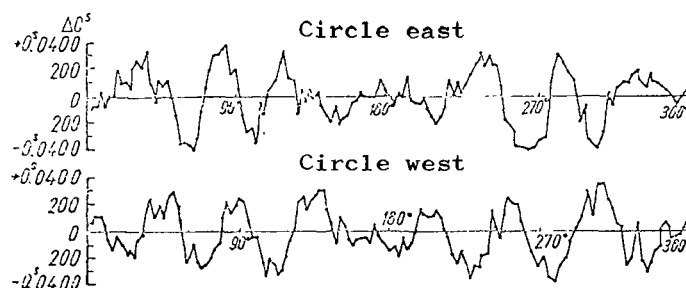


Figure 1.

A comparison with the results of preceding investigations shows that the pivots of the Odessa meridian circle are subject to significant wear.

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CHROMATIC CURVE OF THE OBJECTIVE OF THE AVR-2

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ABSTRACT: The results are presented on the investigation of chromatic aberrations of the refractor AVR-2 object-glass of the Poltava Observatory. The chromatic aberration of position was investigated by three various methods: a) according to the Hartman scheme with application of narrow-band interference filter; b) by photos taken with diffraction grating; c) by photos with an objective prism. The combination of these methods enabled one to carry out the investigation in a wide range of wave lengths as well as to compare the results obtained by three methods different in principle. The value of chromatic aberration is $dS^1 = -0.5 \text{ mm}$ (on the average), the length of the secondary spectrum is $dS_\lambda^1 = 1.4 \text{ mm}$ ($47 \cdot 10^{-5} \text{ F}$). The chromatic difference of magnification was estimated by photos of wide pairs of stars with different spectral classes of pair components.

In this article the results of an investigation of the chromatic aberrations of the objective of the AVR-2 refractor of the Poltava Observatory are expounded, which complete the investigation of its fundamental characteristics which have been published earlier [1].

The investigation of the chromatic aberration of position was carried out by three different methods, the results of which partially overlapped and supplemented each other:

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by the Hartman method with the use of narrow-band interference filters;
from photographs with a diffraction grating;
from photographs with an objective prism.

During the investigation of the objective by the Hartman method [2] it was necessary to prepare a new diaphragm, since the former one (with an aperture of 9 mm) admitted too little light, which would result in excessively long exposures. The diameters of the apertures of the diaphragm were increased to 20 mm, and the number of zones was decreased to five. The distribution of the diaphragms over the zones and position angles is presented in Table 1. The

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TABLE 1.

Zone	D, cm	Position angles			
		I	II	III	IV
1	25	0°		90°	
2	40		45°		135°
3	55	0°		90°	
4	70		45°		135°
5	85	0°		90°	

diaphragm was centered on the mounting so that the main optical axis of the objective passed through the centers of the zones. It is possible to carry out suitable centering with sufficient accuracy by the method expounded in an article by E.A. Gurtovenko [3].

Nine interference filters which admitted the spectral region 442-700 nm were used as reference marks for wavelengths. The characteristics of the filters are presented in Table 2. The filters were mounted in a bushing prepared especially for them which was placed in the plateholder at the place where the operating light filter was usually located.

Initially, a corresponding type of photographic plate was selected for each filter. However, it was soon noted that due to the narrow-band filters it is possible to obtain all the photographic data on panchromatic plates. Since the spectral sensitivity of the latter extends to only 660 nm, it was unfortunately not possible to obtain satisfactory photographs for the last filter, No. 9 ($\lambda = 700$ nm).

TABLE 2.

No. of the filter	λ at maximum transmission, nm	Width of transmission, nm	Coefficient of transmission, nm
1	442	12	40
2	465	10	33
3	484	9	21
4	550	12	40
5	588	10	36
6	593	9	13
7	613	8	13
8	660	29	11
9	700	15	34

We photographed Vega (α Lyrae). The distance between the extra-focal photographs was chosen equal to the maximum separation of the plateholder part of the instrument, i.e., 100 mm. Because of the asymmetry of the position of the focal plane of the objective relative to the ends of the focusing scale, the prefocal images of the diaphragm were obtained approximately twice as large as the extra-focal ones. The distance between the extra-focal photographs was decreased for some of the more dense filters in order not to increase excessively the time of exposure.

The position of the optical center of the plate was shifted somewhat relative to the geometrical center of the plateholder, due to which it proved possible to obtain both images of the diaphragm at the optical center on a single plate after rotating it by 180° . About three or four such plates were obtained /87 for each filter, and six plates for filter No. 1. The measurements and their reduction were carried out by the usual method [2].

The position of the focal plane for each plate was found by means of averaging the separate values of the foci for the five zones. They were provisionally improved in a random respect in the following way. Since the spherical aberration of the objective was already investigated sufficiently well, a corresponding template which represents a distribution curve of the focus values of the objective by zones was prepared on a piece of tracing paper. It was compared with the measured distribution of the foci by zones for each plate. Such a comparison permitted recognition of crude errors in the measured values of the foci of separate zones and so permitted improving them in a random respect. By measuring spherical aberration for different wavelengths with this operation, it is apparently possible to neglect it completely. In Figure 1a the differences ΔF between the values of the foci for the first and fourth zones, where it is a maximum, are plotted along the vertical for all the filters used. No sort of noticeable dependence of ΔF on λ is apparent. These same differences ΔF after improvement of the foci of separate zones are presented in Figure 1b.

All the photographic data was obtained at zenith distances from 23° to 50° and one plate at $z = 77^\circ$. The dependence of the value of the focal length of the objective on the zenith distance of the stars photographed was revealed, namely: the focal length of the objective increases with an increase in z . The relations obtained for the five filters used are plotted in Figure 2, where

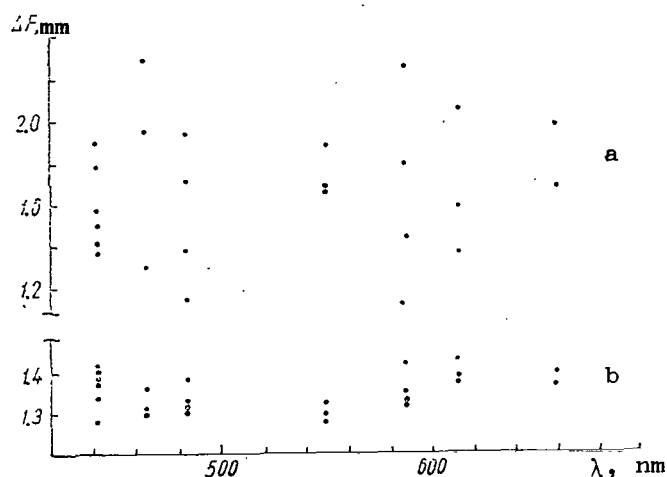


Figure 1.

they are denoted on the figure by various symbols. The distance between the points within the limits of each filter is equal to the difference in the focal lengths of the objective. The combinations of points for various filters relative to the origin of the abscissa axis are plotted arbitrarily. On the basis of the dependence found for the values of the focal length of the objective obtained from all the plates, they were presented for a single zenith distance (30°) and also for an identical temperature at the time of the observations ($+10^\circ$). The curve of chromatic aberration obtained is presented in Figure 3 (1).

The filters permitted an investigation of the behavior of chromatic aberration for 442-660 nm. The absence of the necessary photographic materials deprived us of the possibility of continuing the investigation in the direction of longer wavelengths. To extend it in the direction of shorter wavelengths we used the method of extra-focal photographs with a diffraction grating [4]. The objective was covered by a diaphragm with two circular apertures 40 mm in diameter. The apertures were situated symmetrically with respect to the center of the diaphragm at the identical distance of 60 mm from it. Parallel to the line joining the apertures, the wires of the diffraction grating were stretched. The diameter of the wires was 0.24 mm, the distance between them was 0.26 mm, and the grating constant was $d = 0.5$ mm. /88

Taking extra-focal photographs of some bright star or other (as the latter we again selected Vega) from both sides of the focal plane, we obtain on the plate both two images and a pair of images of the spectra of the first order

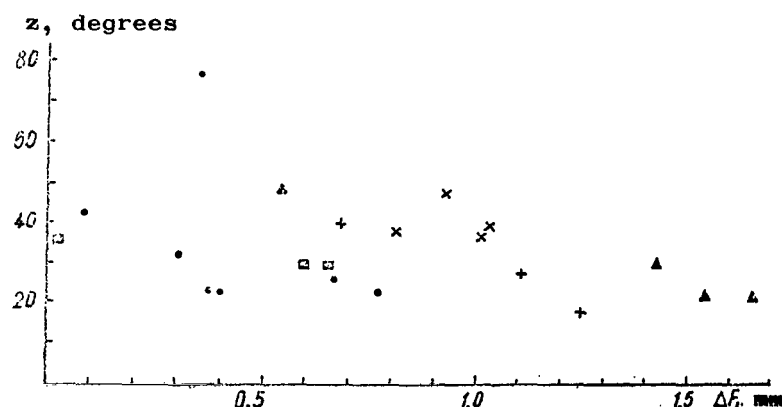


Figure 2.

on both sides of the central image.

The distance between the spectra determines the position of the focal plane F of the objective for the selected wavelength λ . Let these distances be k_1 and k_2 , respectively, at both positions of the plate. Then the position of the focal plane is determined by the expression

$$F_\lambda = F_1 + \Delta F \frac{k_1}{k_1 + k_2}$$

where

F_1 is the reading of the focusing scale at one of the positions of the plate; and

ΔF is the distance between the extra-focal photographs.

The wavelength λ is found from the expression $\lambda = d\iota/F$, where ι is the distance of the selected portion of the spectrum from the central image and F is the focal length of the objective. To simplify the transition from the measured distances ι to λ a table is compiled where λ has been calculated from the equation $\lambda = 0.0001657\iota$.

The chromatic curve was obtained by the method discussed (see Figure 3, 2). The points of the curve, as in the preceding method, are presented for $z = 30^\circ$ and $t = +10^\circ$. As is evident, it was possible to extend the chromatic curve to 388 nm. Further advance in the direction of shorter wavelengths appears to be very inconvenient, since the ends of the spectra corresponding to these λ turned out very smeared, which made accurate sighting on them impossible.

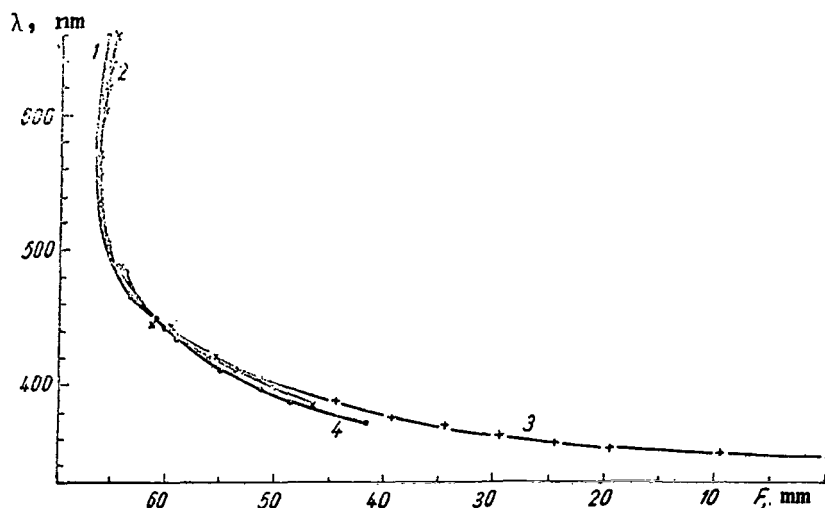


Figure 3.

However, this method gives in principle the possibility of obtaining the position of the focal plane for still shorter wavelengths. For this reason, we decided to modify it in the following way.

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The images of the spectra were photographed at different successive readings of the focusing scale. When a plate appeared to be in the focal plane for some wavelength, the images of the spectra were intersected in the corresponding place. The wavelength for each such point of intersection was found by the usual method, i.e., by determining the distance between this point and the central image.

By such a method it was possible to extend the chromatic curve to 348 nm, utilizing completely the limits of advance of the plateholding part of the instrument but not the potential of the method (see Figure 3, 3).

The proposed method offers the possibility of obtaining the position of the foci only in the case of sufficient slope of the chromatic curve. With the approximation to its peak it is made less effective because of the difficulty of selecting an accurate position of the intersection point. The preceding method, on the contrary, permits obtaining with assurance the positions of the foci near the peak of the chromatic curve.

The chromatic curve of the same type of AVR-2 refractor with our objective of the branch of the Odessa Observatory at Mayaka (curve 4) is also plotted on

Figure 3. It covers the wavelength range of 373-486 nm.

The points of the chromatic curve were determined from "constrictions" in the photographs of the Vega spectra obtained with an objective prism. The easily visible lines of the Balmer series of hydrogen from H_β to H_η were used as the reference wavelengths. (Prior to plotting the curve in Figure 3 its abscissae were corrected for noncoincidence of the zero points of the focusing scales.)

The behavior of the chromatic curve, as is evident from Figure 3, agrees well with the data obtained for the Poltava objective.

The general agreement of the results of the determination of the chromatism of position by the methods discussed is presented in Table 3. The readings of the focusing scale (in mm) obtained respectively with interference filters, a diffraction grating (by the first and second methods) and, finally, an objective prism are denoted by F_1 , F_2 , F_3 , and F_4 . Comparison of the chromatic curves /90

constructed by the three theoretically different methods permits drawing a conclusion about the good agreement among the results obtained.

TABLE 3

λ, nm	F_1 mm	F_2 mm	F_3 mm	F_4 mm	λ, nm	F_1 mm	F_2 mm	F_3 mm	F_4 mm
378			0		443			59.4	
382			9.5		446			61.4	
386			19.5		454		61.6		
388			24.5		455	63.5			
389			29.5		469			63.4	
392			34.5		471		63.1		
393				41.5	484	64.5			
398			39.5		486				64.0
399		46.6			487		64.4		
400				48.5	504		65.4		
400			44.5		520		65.9		
407				51.5	537		66.1		
408			50.0		550	66.2			
408		51.2			554		66.3		
408			51.5		570		66.2		
410				55.0	588	66.5	66.2		
413			53.5		604		65.8		
421		55.7			613	66.0			
423			55.4		621		65.6		
430			57.4		637		65.2		
434				59.0	654		64.8		
438			59.1		660	65.6			
442	60.1								

The amount of chromatic aberration, i.e., the difference of the foci for $\lambda_1 = 486 \text{ nm}$ and $\lambda_2 = 656 \text{ nm}$, for which visual achromats are usually corrected, is $ds' = -0.5 \text{ mm}$ (on the average), which confirms the undercorrected chromatic aberration.

The peak of the chromatic curve occurs in the region around 560-570 nm. The length of the secondary spectrum is $ds'_\lambda = 1.4 \text{ mm}$ ($47 \times 10^{-5} F$). It is usually assumed for objectives of the type investigated to consider that ds'_λ should not exceed $55-65 \times 10^{-5} F$ [2]. From this it is possible to draw the conclusion that the chromatic aberration in position of the objective of the AVR-2 refractor is corrected completely satisfactorily.

Concerning the chromatic aberration of magnification, we decided to restrict ourselves to an estimate of its effect directly on the position of stars of various spectral classes.

With this purpose, four pairs of stars with distances between components of about 40-50' were selected. The stellar magnitudes (visual) were selected to be similar within a pair as far as possible, but the spectral classes, on the contrary, were selected to be as different as possible. The photography was carried out by a method with which the distortion across wide pairs is usually investigated [5], i.e., first one component of the pair was shifted to the center and the other to the edge of the plate, and then the reverse. In the presence of chromatism of magnification the distances between components of a pair are not the same in both of its positions.

The measurements showed that the systematic shifts of stars caused by the effect of chromatism of magnification are insignificant within the limits of the field investigated. The scale for red stars is found to be approximately 1.00005 times greater than for blue stars.

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DETERMINATION OF THE LATITUDE OF THE MAIN ASTRONOMICAL
OBSERVATORY OF THE UKRAINIAN SSR ACADEMY OF SCIENCES
(GOLOSEYEVO)

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ABSTRACT: The article deals with the determination of the longitude of the Goloseyevo Observatory.

The second determination was carried out by the author in 1967, the 2" universal instrument being used.

The local time was determined from observation of star pairs on equal altitudes. The methods of observation and processing are described.

The following values of longitude are obtained: for the observation pillar "Goloseyevo" $\lambda^{\circ} = -2^{\text{h}}02^{\text{m}}0^{\text{s}}.191 \pm 0^{\text{s}}.016$, for the center of the vertical circle $\lambda^{\circ} = -2^{\text{h}}01^{\text{m}}59^{\text{s}}.931 \pm 0^{\text{s}}.016$.

The approximate value of the longitude of the observatory was first obtained in 1950 by the method of measuring the zenith distances of the sun with a 20-second theodolite. A weighted value of the longitude was derived from two clock corrections with an error of $\pm 1^{\text{s}}$. Then to obtain a more accurate value of the longitude observations of near-zenith stars with the Vanshaff vertical circle were used, which were carried out in 1955-1958 during the determination of the constants of its mounting. With this purpose, the results of the observations of star transits carried out by the transit instrument method over the period of six years were processed. From 14 instants of the transit of stars the following value of the longitude of the center of the pier of the vertical circle was obtained:

$$-2^{\text{h}}02^{\text{m}}0^{\text{s}}.013 \pm 0^{\text{s}}.027$$

This value of the longitude was published in the International Directory of Astronomical Observatories "Les Observatoires astronomiques et les astronomes" (by Fernand Rigaux, Brussels, 1959) and also in [1, 7-9].

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With non-rigorous account taken of the inclinations of the horizontal axis of the instrument and the incomplete method of recording the instants of transits, the accuracy of the result proved to be sufficiently high, which confirms the fine stability of the instrument and the high quality of the observations. However, the personal errors of the observer remained unknown. In May-July, 1967, the longitude was determined repeatedly with the help of a 2-second universal instrument. The local time was determined from observations of pairs of stars at various altitudes. The following program was carried out. On May 13, 14, and 15, seven longitudes of the point to be determined in Goloseyevo were obtained from the observations; then, on June 3, 4, 6, and 7, twelve longitudes at the original point in Nikolayevo were observed for the determination of the personal-instrumental difference, after which on June 23, 26, and 28 and July 7, eight more longitudes were obtained in Goloseyevo. A group of eight Zinger pairs was observed at 2-hour intervals of time for the derivation of each value of the longitude. The instants of the star transits were recorded by the five threads of the tube with the help of a recording chronograph by the "eye-key" method. Second signals of the exact time were automatically recorded on the tape of the chronograph. Shifts of the tube in altitude were determined at the start and at the finish of the observation of each star with the help of a Talcott level with a scale division of $\tau = 1''62$. The latitude of the place of observation was known with an error $\Delta\varphi$ not exceeding $\pm 0''.1$. Therefore, rigid selection of the pairs in the groups was not carried out. /92

The error in recording the transit of a star across the horizontal thread was calculated in agreement with the recommendations in [2]. On the average, this error was found to be $m_s = \pm 0''.11$ for the first vertical and $m_s = \pm 0''.05$ for the average instant of transit across the five threads.

The differences in the inclinations of the Talcott level were determined with an error $m_i \simeq \pm 0.5 \simeq 0''.03$.

The apparent places of a large part of the stars observed (more than 90%) were taken on the FK4 system from the Astronomical Yearbook, and from the Catalogue of 2957 Bright Stars for the remaining part (less than 10%), which are presented on the FK4 system. Upon calculation of the coordinates of the stars

the short-period nutation terms were taken into account. The errors in the coordinates of the stars m_* , reduced to the equator, did not exceed the value 0.01.

The mean square error in the determination of time from the observations was calculated for each pair from the equation

$$m_u = \sqrt{\frac{1}{2} (m_s^2 + m_i^2 + m_a^2) \sec^2 \varphi \operatorname{cosec}^2 A'} \quad (1)$$

where

A' is the average angular distance of the star pair from the meridian.

The time corrections u_i were calculated from the equation

$$u = \alpha - \frac{1}{2} (T_W + T_E) + r + \delta_i + \delta_a \quad (2)$$

where

α is the average of the right ascensions of the star pairs;

$1/2 (T_W + T_E)$ is the average moment of observation of a pair;

r is the half-difference of the hour angles of the star pair at the moment of observation;

δ_i is the correction for the difference in inclination of the Talcott level;

δ_a is the correction for the diurnal aberration.

The calculations of the clock corrections were carried out by the standard method in agreement with the recommendations in [2, 3]. The auxiliary tables [4] were used.

The weighted values of the clock corrections $\bar{u}$ were derived for each group. The weights p of the individual values u were taken inversely proportional to m_u^2 . The values of the instantaneous longitudes λ_i were calculated for each $\bar{u}$ from the equation

$$\lambda_i = S - (T + \bar{u}) \quad (3)$$

where

S and T are, respectively, the Greenwich and local sidereal time of the mean instant of observation of a group of pairs.

A weight p_i corresponding to the square of the mean square error M_{ui} of the weighted value $\bar{u}_i$ was attributed to the value of λ_i calculated by such a method.

The quantities $\Delta\lambda_i = \lambda_i - \lambda_m$, $M_{\lambda_i} = M_{ui}$, and p_i are presented in Tables 1 and 2, from which are obtained the weighted values of the instantaneous longitude for Goloseyevo (Table 1) and Nikolayevo (Table 2), and also an estimate of them is made. /93

TABLE 1

1967	$M_{\lambda_i} = M_{u_i}$ (in 0.001)	p_i	$\Delta\lambda_i =$ $\lambda_i - \lambda_m$ (in 0.001)
May 13/14	28	1.28	- 56
May 13/14	22	2.07	- 59
May 14/15	27	1.37	+ 38
May 14/15	28	1.28	+ 67
May 15/16	30	1.11	+ 28
May 15/16	31	1.04	+113
May 15/16	15	4.44	+ 55
June 23/14	26	1.48	- 18
June 23/24	25	1.60	- 16
June 26/27	28	1.28	- 36
June 26/27	23	1.89	+ 14
June 28/29	32	0.98	+ 1
June 28/29	27	1.37	+ 44
July 7/8	15	4.44	+ 27
July 7/8	43	0.54	- 18
26.17			

TABLE 2

1967	$M_{\lambda_i} = M_{u_i}$ (in 0.001)	p_i	$\Delta\lambda_i =$ $\lambda_i - \lambda_m$ (in 0.001)
June 3/4	15	4.44	+ 82
June 3/4	17	3.46	+ 33
June 4/5	15	4.44	+ 12
June 4/5	17	3.46	- 49
June 4/5	11	8.26	- 19
June 4/5	11	8.26	+ 8
June 6/7	16	3.91	- 16
June 6/7	15	4.44	- 22
June 6/7	14	5.10	+ 13
June 7/8	17	3.46	- 3
June 7/8	16	3.91	- 50
June 7/8	14	5.10	+ 7
58.24			

The mean square error of a single determination of the longitude of Goloseyevo is

$$m_{\lambda} = \sqrt{\frac{[\Delta\lambda_i p_i]}{n-1}} = \pm 0^s.066.$$

The mean square error of the weighted value of the longitude is

$$M_{\lambda} = \frac{m_{\lambda}}{\sqrt{[p]}} = \pm 0^s.013.$$

The mean square error in the determination of a single longitude of Nikolayevo is

$$m_{\lambda} = \sqrt{\frac{[\Delta\lambda_i \cdot p_i]}{n-1}} = \pm 0^s.074.$$

The mean square error of the weighted value of the longitude is

$$M_{\lambda} = \frac{m_{\lambda}}{\sqrt{[p]}} = \pm 0^s.010.$$

Then the reductions for the conversion of the instantaneous longitudes to the mean pole were calculated for the average date of the observations. They proved to be insignificant: $0^s.004$ was obtained for May 14, 1967; $0^s.004$ for June 5, 1967, and $0^s.003$ for July 1, 1967. We note that the instantaneous longitude of the derived and original points were obtained by one-way determinations directly from Greenwich. Therefore, the calculations of $\Delta\lambda_0$ were carried out according to the equation

$$\Delta\lambda_0 = \frac{1}{15} (x \sin \lambda - y \cos \lambda) \tan \bar{\varphi} + y \tan \bar{\varphi} \quad (4)$$

where

x and y are the coordinates of the instantaneous pole relative to the average;

$\bar{\varphi}$ is the latitude of Greenwich.

The personal-instrumental difference $\vartheta\lambda$ is obtained from the expression

$$\vartheta\lambda = \lambda_{\text{orig}}^{\circ} - \lambda_{\text{var}}^{\circ} \quad (5)$$

where

$\lambda_{\text{orig}}^{\circ}$ is the longitude of the original point referred to the mean pole;

λ_{var} is the observed longitude of the original point referred to the mean pole.

It proved to be equal to

$$\vartheta\lambda = +0^s.124 \pm 0^s.010.$$

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The final value of the longitude of the specified point in Goloseyevo was obtained from the equation

$$\lambda^0 = \lambda_m + \vartheta\lambda - \Delta\lambda_0 - \omega + \epsilon, \quad (6)$$

where

ω is the correction for the speed of propagation of radio waves and

ϵ is the correction of the cumulative instant of transmission of the signals of standard time in the TI-1 system.

Thus, there is finally derived:

for the Goloseyevo astronomical pier

$$\lambda^0 = -2^h02^m0^s.191 \pm 0^s.016,$$

and for the center of the vertical circle (by means of a geodetic tie-in)

$$\lambda^0 = -2^h01^m59^s.931 \pm 0^s.016.$$

Here, the mean square error of the final value of longitude $M_{\lambda^0} = \pm 0^s.16$ is derived from the equation

$$M_{\lambda^0} = \sqrt{M_{\lambda_m}^2 + M_{\vartheta\lambda}^2} \quad (7)$$

In estimating the accuracy of the final longitude value, the variation in time of the personal-instrumental equation $\vartheta\lambda$ was not known to us, since its determination was carried out only once, approximately at the mean instant of observation of the longitude in Goloseyevo. If the variation of the personal difference $\Delta\vartheta\lambda$ is considered a function of time (as is assumed, for example, in [5]), then it should be considered compensated to a certain extent during the given program of longitude determination. This is of particular importance if one takes into account the brief practically equal intervals of time (22 and 25 days) between the average instants of determination of the personal difference and the longitude. For this reason, we could not use in the estimate of the accuracy of the final longitude value $\Delta\vartheta\lambda = \pm 0^s.020$, which was derived for some average astronomer (see, for example, [5] and [6]), especially since it was derived for a more prolonged time, of the order of six-seven months.

The observatory proved to be more westerly by $0^s.082$ according to the newly-derived value of the longitude of the center of the vertical circle, which for the given latitude amounts to 24.3 m.

It is possible to explain the difference obtained with the former determination by the fact that systematic errors due to the personal difference of the observer and the inclination of the horizontal axis of the instrument were not excluded during observation of the longitude with the vertical circle. A.K. Korol' [1] indicated a possible error because of these reasons of the order of 0.1^s .

In conclusion, it should be wished that the value of the longitude of the Main Astronomical Observatory of the Ukrainian SSR Academy of Sciences be subsequently refined by repeated observations in which the recording of the instants of star transits would be accomplished with the help of a contact micrometer.

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DISCUSSION OF THE RESULTS OF LATITUDE OBSERVATIONS
AT PULKOVO WITH THE ZTF-135
IN THE EXPANDED PROGRAM DURING 1955.0-1961.3

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ABSTRACT: The paper deals with the analysis of the six-year latitude observations with the Freiberg zenith-telescope ZTF-135. The observations were conducted according to an expanded program. The abundant material thus obtained was thoroughly analyzed: a correction to the constant of aberration was derived from the latitude series under investigation, as well as correction to the scale values and its temperature coefficient, declinations and proper motions of the centers of the Talcott pairs.

Errors of star declinations and their proper motions taken from different catalogues were considered in detail.

The estimate is given of the accuracy of the latitude observations at Pulkovo with the ZTF-135 during 1955.0-1961.3. The curve of latitude variations and value of z-term are presented.

Section 1. Basic Data About the Latitude Observations
with the ZTF-135 in 1955.0-1961.3.

The latitude observations with the ZTF-135 in 1955.0-1961.3, as the ob-
servations for 1948-1955, were carried out according to Ya.E. Gordon's program
[1], which was made up for one nutational cycle - from 1948 to 1967. Based on
the suggestion of V.I. Sakharov, the observations in 1955.0-1961.3 were carried
out according to the so-called extended program, i.e., from the setting to the
rising of the sun. Such an undertaking was set up with the aim of continuing
the investigation of short-period variations in latitude previously noted in
the latitudes [2]. During the wintertime in the course of a night, the duration
of the observations in the extended program amounted to 18^h. The group of
observers with the ZTF-135 consisted of four persons.

An extended program of latitude observations is located at Pulkovo for
the second time. The first time the observations on such a program were
carried out in 1915-1929 [2-7]. However, because of an insufficient number
of observers, the density of observations in this latitude series is insigni-
ficantly inferior to the density of observations in 1955-1961.

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TABLE 1

Observer	1955		1956		1957		1958		1959		1960		1961		Total		Mean square errors of latitude observations
	Latitude observations	Results	Latitude observations	Results	Latitude observations	Results	Latitude observations	Results	Latitude observations	Results	Latitude observations	Results	Latitude observations	Results	Latitude observations	Results	
N.M. Bakhrakh	839	56	1086	62	471	26	-	-	-	-	-	-	-	-	2396	144	$\pm 0''.155$
I.F. Korbut	933	57	577	33	-	-	-	-	-	-	21	1	-	-	1531	90	128
L.D. Kostina	-	-	698	47	965	57	1224	61	1741	91	1515	77	409	24	6553	357	151
E.I. Kreinin	-	-	-	-	534	37	1149	63	814	49	-	-	-	-	2497	149	161
N.R. Persiyaninova	1033	64	1116	57	1055	66	1209	64	864	47	715	35	199	14	6191	347	160
S.V. Romanskaya	272	19	192	13	-	-	-	-	-	-	-	-	-	-	464	32	150
V.I. Sakharov	690	44	531	32	-	-	-	-	-	-	197	12	-	-	1418	88	141
N.G. Strelkova	-	-	-	-	-	-	-	-	-	-	609	29	264	19	873	48	178
A.M. Sharavin	-	-	-	-	383	26	902	52	1220	68	1008	57	200	16	3713	219	157
Total	3767		4200		3408		4484		4639		4065		1072		25636		

The other part of the observational work with the ZTF-135 during 1955.0-1961.3 was the derivation of an estimate of a screw revolution of the ocular micrometer from scale pairs and the determination of the instrumental constants on the average three times a year, which is completely sufficient in view of the fine stability of the instrument. (The latter task was carried out primarily by N.R. Persiyaninova (ten times), and the remaining observers participated in this undertaking two or three times each.)

A list of the observers and the quantitative data on the observations during the period 1955.0-1961.3 as well as the mean square errors of the latitude determinations by each observer are presented in Tables 1 and 2. The distribution of the number of clear nights by months and years is given in Table 3.

According to the proposal of V.I. Sakharov, the entire extensive material of the latitude observations during 1955.0-1961.3 and also the observations for 1948.8-1955.0 (about 37 thousand in all) were reprocessed with the use of an accurate processing method under the direction and with the participation of L.D. Kostina [8]. The present paper is a continuation of [8]. Some results of a discussion of a second extended latitude series for 1955.0-1961.3 are expounded in it.

TABLE 2

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Observer	Number of observations of scale pairs						Total	Number of elongation observations in 1961
	1955	1956	1957	1958	1959	1960		
N.M. Bakhrakh	12	16	1	-	-	-	29	-
I.F. Korbut	15	10	2	-	-	-	27	-
L.D. Kostina	-	4	24	271	306	378	983	23
E.I. Kreinin	-	-	9	295	80	-	384	-
N.R. Persiyaninova	20	21	7	257	97	65	467	18
S.V. Romanskaya	6	4	-	-	-	-	10	-
V.I. Sakharov	12	5	-	-	-	-	17	-
A.N. Sharavin	-	-	1	175	217	84	477	30
Total	65	60	44	998	700	527	2394	71

TABLE 3

Month	Number of clear nights								
	1955	1956	1957	1958	1959	1960	Total	1961	Average
January	7	12	10	10	12	14	65	7	10
February	9	17	9	8	13	8	64	7	10
March	17	27	22	20	26	18	130	14	21
April	16	11	17	19	19	18	100	18	17
May	9	21	15	12	19	22	98	-	16
June	16	20	19	14	21	16	106	-	18
July	21	15	17	17	26	22	111	-	18
August	28	16	19	21	20	16	120	-	20
September	18	20	12	19	13	12	94	-	16
October	17	15	15	11	9	7	74	-	12
November	8	1	10	13	8	6	46	-	8
December	12	5	12	10	6	3	48	-	8
Total	178	180	177	174	192	162	1063		

Section 2. Determination of the Correction for the Constant of Aberration

1. The correction for constant of aberration is determined from the latitude observations from the discrepancy obtained as a result of the external smoothing of the latitudes by the chain method. Strictly speaking, other causes (we recalled that the corrections for oscillation of the vertical, the Oppolzer term, and other corrections were already taken into account by us in the calculation of the latitudes) can have an effect on the size of the discrepancy, for example, the diurnal variations in latitude, if they exist. To reduce the effect of the diurnal latitude variation there should be a program of latitude observations consisting of 24-hour groups, and smoothing should be carried out by hour groups. The duration of the observations on each night should amount to 4 hours, and the observations should be carried out symmetrically relative to midnight. The accidental error in the determination of the discrepancy decreases because of the large number of observations used.

A.A. Nemiro [9] proposed a method which offers the possibility of determining the constant of aberration from latitude observations based on the Talcott method, free from the effect of the diurnal term. This method is based on the fact that the inaccuracy in the accepted value of the aberrational

constant affects in various ways the latitudes determined from pairs which include stars situated at various zenith distances. For this, a program of latitude observations is required with zenith distances up to $60-70^\circ$. And it is necessary to carry out the observations on an extended program which includes the evening and morning hours.

2. The inaccuracy in the value accepted for the revolution value of the screw of the ocular micrometer. Usually, prior to the start of observations of a latitude program, the value of a revolution is known with sufficient accuracy, and its small inaccuracy is not reflected in the size of the discrepancy. If the inaccuracy in the revolution value is not very small, then it can exhibit an effect primarily in that case where the right ascension in the center of the group with which the smoothing is started is equal to 0^h or 12^h . In this case, in a year at the end of the closed cycle the average of the half-differences in zenith distances of the given group changes by 0.3 , i.e., by approximately half a screw revolution for zenith-telescopes of the usual sizes. For example, an inaccuracy in the revolution value of 0.2 should give an amount 0.1 in the discrepancy.

During observations on an extended program, when each tie-in of two 2-hour groups is observed four months, a small difference can appear in the average values of the half-differences of the zenith distances of pairs of one and the same group (near $\alpha = 0^h$ and 12^h) in two adjacent tie-ins. This is possible if the observations of the adjacent tie-ins were carried out (because of the weather conditions) such that the preceding tie-in of the groups was observed at the start of its period of observations and the succeeding one at the end of the period of observations (in the course of a month $\Delta z_{\text{avg}}/2$ of a group around 0^h or 12^h in right ascension changes by approximately $|0.03|$). This cause can have an effect during observations on a 4-hour program if the inaccuracy in the value of a revolution is especially large.

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3. By far more significant is the effect of a variation in the value of a screw revolution during an annual cycle. It is easy to show that if a small change in the value of a revolution occurs linearly, then the effect of a change in the screw revolution value on the size of the discrepancy is approximately equal to the product $\Delta \times (12/11) M_1$, where Δ is the change in the

value of a screw revolution over the annual cycle and M_i is the averaged value of the half-differences in zenith distances of the pairs in a group with which the external smoothing is started and concluded. In order that the change in the value of a screw revolution not affect the size of a discrepancy upon the determination of the correction to the constant of aberration, the smoothing should be begun from the group for which the average value of the half-distances of zenith distances of pairs is close to zero. It is necessary to neglect part of the observations. Perhaps one should from this point of view review all the determinations of the constant of aberration from the discrepancies obtained during the smoothing of latitude observations.

4. The inaccuracy in the value accepted for the temperature coefficient of the screw of the ocular micrometer. The effect of this inaccuracy can be estimated from the equation

$$\sum_{i=1}^{12} \Delta \alpha M_i (t_i - t'_i) = \Delta \varphi_{\Delta \alpha}. \quad (1)$$

Here

$\Delta \alpha$ is the inaccuracy in the temperature coefficient of the screw of the micrometer;

M_i is the average value of the half differences of the zenith distances in the group;

t_i and t'_i are the average temperatures of the evening and the morning observations of the corresponding groups.

5. The inaccuracy in the proper motions of the centers of the Talcott pairs. The inaccuracy in the average value of the proper motions of the centers of Talcott pairs of the group with which the external smoothing is begun and concluded can in particular have an effect on the value of the discrepancy, since the observations of this group separate the interval into one year.

The determination of the correction to the constant of aberration was carried out with the equation

$$r + A \Delta k = 0, \quad (2)$$

where r is the discrepancy

$$A = \frac{\sum_{i=1}^{12} [(Cc' + Dd')_i - (Cc' + Dd')_{i+1}]}{20.496}. \quad (3)$$

For Pulkovo, the coefficient $A = 4.1$ from the observations of the extended program, and during observation on the 4-hour program in 1948.8-1955.0 this coefficient turned out to be equal to 4.6, i.e., the coefficient A increased during observation around midnight. Thus, the inaccuracy in the constant of aberration during latitude observations with the ZTF-135 at Pulkovo enters into the size of the discrepancy with a factor of 4.

The constant of aberration from the observations in 1955.0-1961.3 is equal ^{/99} to 20".482, and from the latitude series of 1948.8-1955.0, it was found equal to 20".487. We note that each of these values was obtained as the average of the two versions of determining the discrepancy: from the combined smoothing of latitudes for 6 years (for 1955.0-1961 - 5 years), in which the inaccuracy in the value of a screw revolution of the micrometer was provisionally excluded, and from latitude smoothing carried out separately for each year.

The correction to the constant of aberration obtained from the discrepancy of the smoothing of the latitudes determined according to the Talcott method is proportional to $\sin \varphi$ [10], where φ is the latitude of the place. Therefore, the more northerly the observatory is located, the somewhat more certainly should the corrections to the constant of aberration be obtained, since the accidental errors in the differences of the group mean values of the latitudes are divided by a large coefficient.

A summary of the determinations of the constant of aberration carried out primarily during the last 15 years based on material of observations over many years at the Pulkovo Observatory, starting from 1840 and with various instruments, is presented in Table 4.

TABLE 4

Instrument	Constant of aberration	Literature source
Transit instrument on the 1-meter vertical (1840-1842)	20".486 $\pm$ 0".006	[11]
Large transit instrument (1845-1885)	20.488 $\pm$ 0.018	[9]
Zenith-telescope (ZTF-135) (1915-1941)	20.512 $\pm$ 0.003	[12]
Zenith-telescope (ZTF-135) (1920-1928)	20.511	[13]
Zenith-telescope (ZTF-135) (1929-1941)	20.511 $\pm$ 0.007	[14]
Photoelectric transit instrument (1948-1950)	20.484 $\pm$ 0.021	[15]
Polar tube (1953-1959)	20.496 $\pm$ 0.0099	[21]

Since a fairly accurate value of the constant of aberration is assumed during the processing of latitudes, it is possible to have confidence that it does not have an effect on the investigation of the diurnal term.

Section 3. Determination of the Correction for the Temperature Coefficient of the Ocular Micrometer's Screw

To determine the correction for the temperature coefficient of the ocular micrometer's screw ($\Delta\alpha$) the results of latitude observations were used. To this end, the differences were set up between the latitudes determined from two Talcott pairs of the program closely situated in time, chosen so that the differences in the half-differences of zenith distances of these close pairs would be sufficiently large. We derive an equation for determining the correction for the temperature coefficient of the screw.

$$\left. \begin{aligned} \varphi_0 &= \varphi_i + \Delta\delta_i + \frac{\Delta z_i}{2} \Delta R_0 + \frac{\Delta z_i}{2} t_i \Delta\alpha && \text{for the } i\text{-th pair} \\ \varphi_0 &= \varphi_m + \Delta\delta_m + \frac{\Delta z_m}{2} \Delta R_0 + \frac{\Delta z_m}{2} t_m \Delta\alpha && \text{for the } m\text{-th pair,} \end{aligned} \right\} \quad (4)$$

$$\varphi_m - \varphi_i = \Delta\delta_i - \Delta\delta_m + \frac{\Delta z_i}{2} \Delta R_0 - \frac{\Delta z_m}{2} \Delta R_0 + \frac{\Delta z_i}{2} t_i \Delta\alpha - \frac{\Delta z_m}{2} t_m \Delta\alpha. \quad (5)$$

The average difference $\Delta\varphi_{\text{avg}}$, which is then subtracted from each individual difference of the given combination of pairs, is determined from all the dates for each combination of two pairs. A constant part is included in these differences, notably: the differences in the corrections for the declinations and $\angle 100$ the differences in the half-differences of zenith distances of the Talcott pairs, which within half a year (the limiting duration of observation of a Talcott pair in an extended program) can be considered constants.

The equations for determining the corrections to the value of a revolution have the form:

$$\Delta\varphi_{m,i} - \Delta\varphi_{\text{avg}} = \left(\frac{\Delta z_i}{2} t_i - \frac{\Delta z_m}{2} t_m \right) \Delta\alpha - \left(\frac{\Delta z_i}{2} t_i - \frac{\Delta z_m}{2} t_m \right)_{\text{avg}} \Delta\alpha \quad (6)$$

and are solved by the method of least squares.

For determining $\Delta\alpha$ 19 combinations of differences of pairs from the observational material in 1955, 1957 and 1959 were used. The correction to the value of the temperature coefficient (-0.0004) assumed in processing latitudes of 1955-1961.3 was found equal to $+0.00016$ for one revolution.

The procedure for determining the correction to the temperature coefficient of the screw, the combinations of pairs, and their number were taken the same as by V.I. Sakharov in determining the correction to the temperature coefficient from observational material of 1948.8-1955.0.

Section 4. Determination of the Correction to the Value of a Screw Revolution of the Ocular Micrometer and the Corrections to the Declinations and Proper Motion of the Centers of Talcott Pairs

During the smoothing of latitudes by the chain method with the purpose of determining the corrections of the latitudes knowledge of a very accurate value of a screw revolution R is not required beforehand. But in this method, as is well known, it is assumed that the value of a screw revolution does not vary during the annual cycle of observations. Therefore, we gave a good deal of attention to its determination.

The value of a screw revolution was determined by several methods:

from internal smoothing of the latitudes,

from external smoothing of the latitudes,

from observations of scale pairs, and

from observations of near-polar stars in elongation.

During processing of the latitude series of 1955.0-1961.3, the values of a screw revolution obtained from internal smoothing of the latitude for each year were assumed. Such a method of determining the value of a screw revolution was proposed by V.I. Sakharov and first applied to Pulkovo observations in the paper [16].

Because of the large errors in the declinations of the stars comprising a Talcott pair, the values of ΔR determined for each group are very different. At the same time, each group exhibits from year to year one and the same nature of variation of the revolution value. Therefore, in concluding one annual closed cycle of observations, it is possible to compare ΔR determined from the group with which the annual cycle began and concluded.

For sure control of the behavior of the screw revolution value, it should be determined each time from the internal smoothing of the latitudes after ob-

servations of each succeeding connection of groups and comparing the values obtained.

A program of 36 scale pairs made up by E.I. Kreininiiy was intensively observed in 1958-1960 with the ZTF-135. The positions and proper motions of the scale pairs involved were taken during the processing of the observations from the GC catalogue. The results of the determination of the revolution value from separate pairs shows a large inaccuracy in the GC catalogue; from some pairs the revolution value is systematically overestimated, and from others, on the contrary, it is underestimated. Therefore, the results obtained for the determination of the revolution value of the ocular micrometer for the present are of value only for judging the constancy or variability of the screw revolution value. In particular, the annual mean values of R in 1958, 1959, and 1960, determined from scale pairs, agree well among themselves. The values of the corrections to the revolution value obtained from the internal smoothing of the latitudes for this same period can also be considered practically identical.

We propose in the future to recalculate the observations of the scale pairs using contemporary catalogues, for example, AGK 3 - taking into account the new value of the constant of aberration and the revision of the astronomical year books. It is more suitable to use for this purpose the AGK 3 catalogue because its epoch of observations (approximately from 1958) is close to the epoch of the observations of the scale pairs and the errors in the proper motions do not noticeably affect the results of the determination of R.

At the conclusion of the observations of the extended program, the screw revolution value was determined from observations of near-polar stars at elongation, the program of which was made up by L.D. Kostina. From 71 observations of elongations, a screw revolution value was obtained which agrees to $0^{\circ}001$ with the determinations of R from the internal smoothing of the latitudes.

The accuracy of a single determination of a screw revolution value from elongations in 1961 is equal to $\pm 0^{\circ}015$. In 1967 during our last determinations of the revolution value from elongations, this accuracy became equal to $\pm 0^{\circ}010$.

The method of elongations allows the determination in a comparatively

short time of the revolution value with great accuracy. If the accuracy of a single determination of the screw revolution value is equal to ± 0.01 , then it is possible to obtain the revolution value with an accuracy of ± 0.001 from 100 elongations. For our program it is possible to observe on the average three elongations in 1 hour; consequently, to observe 100 elongations it is necessary to spend 33 hours of straight observational time. The internal accuracy of the observations is significantly increased if near-polar stars are observed through half a screw revolution.

In order to decide which method - scale pairs or elongations - gives the best result, we propose to make up a program of observations such that it will be possible to observe during the long winter nights (from the end of September to the middle of March) both near-polar stars in elongations and scale pairs comprised of those same near-polar stars. The plan of the observations is the following: during the first half of the night - observations of the near-polar stars at eastern elongation; during the second half of the night - observations of the scale pairs of these same stars; during the remaining time of the year - in the first half of the night observations of scale pairs of near-polar stars; in the second half of the night observations of the same near-polar stars at western elongation.

If the elongations follow one another rapidly, it is then possible to observe part of the elongations on one evening and the rest - on another. Unfortunately, there is a small number of sufficiently bright near-polar stars in the northern sky. It might be possible to solve the problem posed better with the help of the ZTL-180, which are faster than zenith-telescopes of standard sizes. However, even this does not answer the question of a better method of observations, since the accuracy of the determination depends not only on the accuracy of the declinations but also on the difference in the declinations of the pairs involved, and for near-polar stars - on the value of the declination. It would be possible to determine this dependence, deriving R from the scale pairs comprised of near-polar stars with different differences in their declinations and from elongations of near-polar stars with different declinations.

It is possible to achieve good results with any method of deriving R which has a large number of investigations. The question is with which method

the best result is achieved with a minimum expenditure of efforts on the ob- /102
servations (the question of computer processing is not particularly significant
now). To obtain R_{elong} with an accuracy of $\pm 0''.001$ (with an error of unit weight
of $\pm 0''.01$) requires 33 to 34 hours. How much time is required to obtain R with
the same accuracy by another method? It is advantageous to observe wide-
pairs on the ZTL-180 zenith-telescopes, since it is possible to set up such
pairs even from stars with large declination errors so that the error of unit
weight in the determination value is less than $\pm 0''.01$.

However, we note that it is necessary to determine the revolution value of
the ZTL-180 with a high degree of accuracy since the revolution value of its
ocular micrometer is less than on zenith-telescopes of standard sizes. To
compare the accuracies of the determination of R of instruments with a different
value of the screw revolution value it is necessary to compare the errors in the
determination of the unit weight (i.e., of a single determination) made in the
screw revolution value of a given instrument. In the observation with ZTL-180
instruments of latitude programs comprised of Talcott pairs with large differ-
ences in zenith distances, the requirement for accuracy in the determination of
the screw revolution value of the ZTL-180 is increased still further.

If it would be possible to select sufficient scale pairs with well-deter-
mined declinations, it would be possible to confirm that the method of scale
pairs is more economical than the method of elongations. V.I. Sakharov carried
out a calculation for normal zenith-telescopes (a field of view of $25'$), based
on the following conditions. If the declination errors in the catalogues are
taken to be $\pm 0''.30$ and the differences in the declinations of the scale pairs as
 $20'$, then it is sufficient for two seasons (for example, fall and winter) to
have programs which contain about 100 scale pairs and observe each pair about
two times to obtain the screw revolution value with an accuracy of $\pm 0''.0010$.

The corrections to the declinations of the centers of the Talcott pairs
are obtained in two versions:

from the smoothing separately of the latitudes for each year and by
averaging the results obtained:

from a combined smoothing of the latitudes for five years of observations.

In the second method, the inaccuracies in the screw revolution value were first excluded from the latitudes determined from the internal smoothing of the latitudes in each year. In view of the fact that, in 1957, it was not possible to obtain a closed cycle of observations, which is necessary for the determination of the external corrections to the declinations, the observations of this year were not used in determining the corrections to the declinations. The accuracy of the determination of corrections to the declinations was obtained from a comparison among themselves of the corrections of the declinations obtained in each of the five years of observations. The average value of the mean square error in the determination of the correction to the declinations was equal to $\pm 0''026$, which is somewhat lower than the accuracy of the series 1948-1955 ($\pm 0''023$). This is probably explained by the fact that in 1955.0-1961 the data used were for only five years.

We compared the declinations and proper motions obtained for the Talcott pairs with the newest catalogues and gave an evaluation for some of them from the results of this comparison. The results obtained will be published later.

The corrections to the latitudes were obtained from the equation:

$$\Delta\varphi_i = \Delta\delta_i + \frac{\Delta z_i}{2} \Delta R, \quad i=1, 2, \dots, 102, \quad (7)$$

where the ΔR were determined from the internal smoothing of the latitudes and the values obtained from the combined reduction of latitudes for five years were taken as $\Delta\delta_i = \Delta\delta_{\text{int}} + \Delta\delta_{\text{ext}}$.

Using the data of the internal smoothing, we carried out an evaluation of the accuracy of the latitude observations by the method proposed by V.I. Sakharov. This method of evaluating the accuracy of the latitude determination was applied earlier in the paper [16]. The average values for six years of the mean square errors in the determination of latitudes of all the observers are presented in the last column of Table 1. The mean square error for all the observers is $\pm 0''154$ - a quantity which characterizes the average error in the determination of a single latitude in 1955.0-1961.3. /103

Section 5. Curve of the Latitude Variation of Pulkovo and the z-term from the Observations of 1955.0-1961.3.

After obtaining the smoothed individual latitudes normal points of 250 latitudes were formed (the mean square error of its determination was about

±0"01), with which there was constructed by means of analytical smoothing the curve of the latitude variations of Pulkovo. The smoothing was carried out by Whittaker's method [17] for the case of measurements of unequal accuracy approximately the same as in [18]. In our paper, we used the results obtained by V.I. Sakharov as a result of his investigation of this method. V.I. Sakharov showed that it is possible to use the equations of the method indicated for the case of unequally-spaced ordinates, if the deviations of the individual intervals for corresponding ordinates from the average interval conform to normal law (we obtain for the characteristic of the ordinates of a given latitude series $p(\chi^2) = 0.74$). In this case, compensation of the computed errors occurs because of the inexact uniform spacing of the ordinates. The results of a calculation of smoothed ordinates are obtained just as if the latter condition were rigidly fulfilled. This frees us from applying a more complicated program of calculation of the smoothed ordinate values based on equations of general form with the so-called divided differences. The optimal smoothing parameter ϵ was determined for a given concrete case by the method proposed by V.I. Sakharov. The smoothing was carried out on the M-22 computer with M.S. Chubeya's program made up at the request of V.I. Sakharov.

A detailed investigation of the method of analytic smoothing was expounded in the article of V.I. Sakharov and M.S. Chubeya.

The data on the oscillation of Pulkovo's latitude from observations with the ZTF-135 during 1955.0-1961.3 (unsmoothed and the smoothed normal latitude points) are presented in Table 5. Gaps in the column labeled $\Delta\varphi_{\text{unsm}}$ indicate that at the given place there are no unsmoothed latitude values, and the smoothed values were determined with a weight $p = 0$, as was proposed by I.I. Glagoleva [18].

With the help of the curve thus obtained for the variation in latitude the mean latitudes of Pulkovo were calculated, which were determined from the equation of V.I. Sakharov [16]:

$$\psi_{t+9} = \frac{1}{40} \sum_{t=1}^{t=5} (\varphi_t + \varphi_{t+3} + \varphi_{t+5} + \varphi_{t+8} + \varphi_{t+9} + \varphi_{t+11} + \varphi_{t+14}). \quad (8)$$

A harmonic with a period of 0.6 year was inserted into the equation for the best approximation of the Chandler wave. The mean value of the latitude from the second six-year series was equal to $\psi_0 + 0''.007$. The data on the variation

TABLE 5

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 $\varphi = 59^{\circ}46'15''.00 + \Delta\varphi$

Point No.	τ	n	$\Delta\varphi_{unsm}$	$\Delta\varphi_{sm}$	Point No.	τ	n	$\Delta\varphi_{unsm}$	$\Delta\varphi_{sm}$
1	1955.07	249	0".58	0".60	57	38	241	32	33
2	16	249	68	65	58	46	247	43	41
3	20	242	66	67	59	53	233	48	49
4	25	295	66	67	60	60	230	60	60
5	33	243	68	63	61	65	259	70	68
6	41	239	58	58	62	68	261	75	74
7	49	263	50	53	63	74	269	74	77
8	56	254	47	43	64	82	272	77	77
9	61	271	47	45	65	88	227	74	74
10	64	245	41	42	66	93	280	72	68
11	67	254	41	40	67	1959.01	234	59	60
12	72	257	40	38	68	09	239	49	51
13	81	231	33	36	69	15	245	42	44
14				36	70	18	270	39	38
15	91	255	37	36	71	19	267	34	35
16	96	269	37	39	72	22	277	33	32
17	1956.06	255	42	42	73	25	252	33	30
18	13	281	47	45	74	1959.29	270	28	28
19	16	241	48	49	75	33	242	23	25
20	20	279	53	52	76	37	230	23	24
21	21	265	55	55	77	42	240	23	24
22	23	252	56	58	78	48	221	26	26
23	1956.28	256	61	61	79	53	257	28	30
24	35	249	65	64	80	67	267	40	37
25	40	246	67	66	81	62	280	45	44
26	44	256	64	67	82	68	263	49	51
27	52	246	67	67	83				53
28	61	241	67	67	84	79	242	63	63
29	68	260	64	64	85	87	206	68	67
30	71	277	61	60	86	91	267	70	69
31	76	263	55	54	87				69
32	82	224	49	47	88	1960.04	255	67	67
33				40	89	09	241	63	64
34	96	59	26	33	90	15	271	60	61
35	1957.05	164	26	28	91	20	256	57	58
36	14	215	28	26	92	24	249	59	54
37	19	254	27	26	93	27	266	48	50
38	22	249	28	28	94	32	258	45	46
39	25	265	34	33	95	38	253	45	42
40	32	230	39	40	96	43	256	40	39
41	38	252	47	50	97	50	252	34	37
42	46	250	58	60	98	56	263	36	38
43	55	236	71	68	99	60	256	44	40
44	61	231	76	74	100	1960.65	258	43	44
45	67	217	76	75	101	72	264	47	47
46	77	261	72	71	102	78	233	52	51
47	85	223	63	64	103	86	212	54	54
48	93	255	55	54	104				57
49	1958.00	268	44	43	105	1961.05	154	56	58
50	09	303	28	33	106	14	246	62	59
51				26	107	21	236	57	59
52	19	352	23	22	108	27	256	57	57
53	23	255	21	20	109	32	248	56	55
54	25	260	23	21	110				52
55	27	260	22	23	111	43	259	50	50
56	31	243	26	27					

TABLE 6

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Year	$\Delta\psi_i = \psi_i - \psi'_0$ (In O!001)									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1955	—	—	—	—	—	—	—	(+ 8)	(+ 9)	+ 9
1956	+10	+11	+ 9	+ 7	+ 7	+ 5	+ 4	+ 1	— 2	— 3
1957	— 7	— 9	—11	—13	—14	—13	—14	—14	—11	—10
1958	— 9	— 5	— 5	— 3	— 1	— 1	— 1	+ 1	— 1	— 1
1959	— 1	— 3	— 4	— 4	— 5	— 5	— 3	— 1	+ 2	+ 6
1960	+ 9	+13	+17	+19	+21	+22	—	—	—	—

Note: ψ'_0 is the average value of the mean latitudes from the observations for 1955.0-1961.3; ψ_i is the average latitude for each tenth of a year.

TABLE 7

Year	$\Delta\varphi_i = \varphi_i - \psi_i$ (In O!01)									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1955	+ 6	+13	+17	+15	+ 9	+ 2	— 4	—12	—15	—15
1956	—12	— 6	+ 3	+11	+15	+17	+17	+11	— 2	—12
1957	—19	—23	—22	—11	+ 4	+16	+19	+20	+19	+ 8
1958	— 6	—19	—29	—25	—15	— 4	+10	+25	+27	+22
1959	+12	— 1	—15	—23	—26	—23	— 9	+ 3	+14	+18
1960	+18	+13	+ 5	— 4	—11	—15	—10	— 4	+ 2	+ 6
1961	+ 8	+ 9	+ 9	+ 6	+ 4	—	—	—	—	—

TABLE 8

Year	z-term (In O!01)									
	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1955	0	—1	0	—1	—2	—1	+2	+ 1	+2	+2
1956	+1	0	0	—1	—4	—4	—2	0	0	+3
1957	+6	+4	0	0	+1	+2	—6	—10	—5	—2
1958	+1	+4	0	+4	+5	+1	—1	0	—3	—3
1959	—1	—1	—1	0	0	—3	—2	— 5	—5	—3
1960	0	+5	+7	+7	+3	—1	+2	+ 2	+3	+2
1961	+1	0	—1	—4	—2	—	—	—	—	—
Mean	+1	+2	+1	+1	0	—1	—1	— 2	—1	0

of the mean latitude are presented in Table 6. The amplitude of the variation of the mean latitude is equal to $0^{\circ}037$; for some years the mean latitude is systematically less, and sometimes it is systematically greater than its average value. With these values of the mean latitude Ψ_i the variations in the latitudes of Pulkovo were obtained for each tenth of a year, $\Delta\varphi_i$. These data are presented in Table 7.

To determine the z-term of the extended latitude series, we compare our observations $\Delta\varphi_i$ with the curve of the variation in Pulkovo's latitude calculated from the coordinates of the pole which were determined by N.I. Panchenko (up to 1957.4) [19], by A.A. Korsun' (up to 1960) [20], and by the International Time Bureau at Paris (from 1960). The data on the z-term are presented in Table 8. It is evident from the table that the z-term at Pulkovo is not stable. The greatest oscillation of the z-term is noted in 1957. The values of the z-term averaged over six years have an amplitude of $0^{\circ}04$. The z-term will be investigated in greater detail by us in a succeeding paper.

In conclusion, I express my gratitude to the section's laboratory assistants G. A. Vinogradova, O. B. Bryushkova, and T. D. Kudermetova for calculations during the preparation of this paper and also V. I. Sakharov and M. S. Chubeya for calculations of the smoothed normal latitude points.

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Translated for the National Aeronautics and Space Administration
under contract NASw-2038 by Translation Consultants, Ltd.,
944 S. Wakefield St., Arlington, Virginia 22204.

☆ U. S. GOVERNMENT PRINTING OFFICE : 1972 720-403/324



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